TIME VARYING INCREASING DEMAND INVENTORY MODEL FOR DETERIORATING ITEMS

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ABSTRACT
An inventory model for deteriorating items is considered in which demand increases with respect to time, deterioration rate, inventory holding cost and ordering cost are all continuous functions of time. Shortages are completely backlogged. The planning horizon is finite. The optimal replenishment policy and decision rule, which minimizes the total cost, is developed. A numerical example is given to illustrate the derived model. Sensitivity analysis is presented for the model.

Key words: Increasing demand, deteriorating items, deterministic inventory model.

MSC: 90B05

RESUMEN
Un modelo de inventario para los artículos deteriorables en los que aumenta la demanda con respecto a tiempo, razón de deterioro, el costo de mantenimiento del inventario y el costo de las órdenes son todas funciones continuas del tiempo. Las escaseces son completamente satisfechas. El horizonte de la planificación es finito. Se desarrollan la política del reabastecimiento óptimo y la regla de decisión que minimizan el costo total. Un ejemplo numérico se brinda para ilustrar al modelo derivado. Se presenta el análisis de sensibilidad del modelo.

1. INTRODUCTION

In deterministic inventory models, it is assumed that demand rate is constant and optimal replenishment policy depends on ordering cost; inventory carrying cost and shortage cost. Many researchers are engaged in extending Wilson's economic order quantity (EOQ) model to consider time varying demand patterns. The pioneer work was done by Donaldson (1977) who developed an EOQ model with linearly increasing demand over a finite time horizon. Wagner and Whitin (1958) gave Dynamic Programming (DP) algorithm for the determination of an EOQ by treating time to be a discrete variable. Silver (1979) obtained a simple solution for Donaldson (1977)'s problem using the Silver-Meal (1969) heuristic. Ritchie (1984) developed the exact solution for a linearly increasing demand, for Donaldson (1977)'s problem. Mitra, Cox and Jesse (1984) presented an algorithm for adjusting the EOQ model for the case of demand patterns having linearly increasing or decreasing trends.

Deb and Chaudhuri (1987) extended the inventory lot-sizing problem with linearly increasing demand by allowing shortages. It was a heuristic method to determine the economic replenishment policy. Murdeshwar (1988) tried to develop the exact analytic solution to the problem of Deb and Chaudhuri (1987). Goyal (1988) and Dave (1989) pointed out error in the cost function of Deb and Chaudhuri (1987). Goswami and Chaudhuri (1991) reconsidered same problem under the assumption of finite rate of replenishment. Datta and Pal (1992) developed a mathematical model assuming that replenishment intervals are in arithmetic progression.

The above discussed models are developed under the assumption that an item in an inventory system remains intact. But gradual physical decay in course of time or direct spoilage are unavoidable circumstances in reality. Hence it should be considered in inventory modeling. Dave and Patel (1981) developed an inventory model for deteriorating items with time proportional demand, instantaneous replenishment and no shortages. This model was extended by Sachan (1984) by allowing backlogging. Chung and Ting (1993) proposed a heuristic model for replenishment of deteriorating items with a linear trend in demand whether or not a planning horizon exists. These models are derived under the assumption that items deteriorate at a constant rate. Covert and Philip (1973) gave EOQ model for a variable rate of deterioration by assuming a two-parameter Weibull distribution. Philip (1974) used a three parameter Weibull distribution for the deterioration time while Goswami and Chaudhuri (1992) considered the deterioration rate to be time-proportional. A survey of literature on inventory models for deteriorating items was given by Raafat (1991).

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In above discussed models, the inventory holding cost per item per unit time and ordering cost order are taken to be known constants, which is an unrealistic situation. Researchers like Van der Veen (1967), Weiss (1982), Naddor (1966), Goh (1994) derived EOQ models with different functional forms of holding cost. In the present paper we have developed a generalized EOQ model for deteriorating items where demand is a linearly increasing function of time where as deterioration rate, holding cost and ordering cost are linearly increasing function of time, which is justified when the price index increases with time. Shortages in inventory are allowed and completely backlogged.

2. ASSUMPTIONS AND NOTATIONS

The assumptions of the model are as follows:

1. An inventory system of a single item operates for a prescribed time-horizon of length H.

2. The demand rate $R(t) = a + bt$, $a > 0$, $b > 0$ $a >> b$ is increasing function of time.

3. Shortages are allowed and are completely backordered. Shortages are not allowed in the last replenishment cycle. The shortage cost is π per unit short per unit time.

4. C is the purchase cost/unit.

5. The holding cost, h(t) per unit per time unit is time dependent and (say) its functional form is

$$h(t) = ht + \alpha t,\ h > 0,\ \alpha \geq 0.$$

6. The ordering cost, K(t) depends on the total time elapsed up to the beginning of each cycle and is taken as

$$K(t) = K + \beta t,\ k > 0,\ \beta \geq 0.$$

7. The units in an inventory deteriorate at a constant rate $\theta, (0 \leq \theta < 1)$ per time unit.

8. There is no repair or replacement of the deteriorated units during the planning horizon H.

9. m - orders placed during the time-horizon H.

10. The inventory level at the end of the time-horizon H is zero.

11. Replenishment rate is infinite and lead-time is zero.

12. S_j is the time point at which the inventory level in the j^{th}-replenishment cycle drops to be zero, $j = 1,2,\ldots,(m - 1)$. For the last replenishment cycle $t_m = H$.

13. $t_j = \frac{jH}{m}$, $j = 0,1,.....,m$, is the total time elapsed and including the j^{th} - replenishment cycle.

3. THE MATHEMATICAL MODEL

Let $Q_j(t)$ $(j = 1,2,........,m)$ be the inventory level at any time t in the j^{th} - replenishment cycle. The differential equation governing instantaneous state of $Q_j(t)$ is given by

$$\frac{dQ_{1j}(t)}{dt} + \theta\, Q_{1j}(t) = a + bt,\ t_{j-1} \leq t \leq S_j,\ j = 1,2,\ldots,m \tag{1}$$

With the boundary condition $Q_{1j}(S_j) = 0$, and

$$\frac{dQ_{2j}(t)}{dt} = a + bt,\ \ S_j \leq t \leq t_j,\ \ j = 1,2,\ldots,(m-1) \tag{2}$$

with $Q_{2j}(S_j) = 0$

The solution of (1) is given by

$$Q_{1j}(t) = \frac{1}{\theta^2}\left\{(\theta(a+bS_j)-b)e^{\theta(S_j-t)} - \theta(a+bt)+b\right\} \quad (3)$$

$$t_{j-1} \le t \le S_j,\ j = 1,2,\ \ldots,m$$

and that of (2) is

$$Q_{2j}(t) = a(t - S_j) - \frac{b}{2}(S_j^2 + t^2) \quad (4)$$

$$S_j \le t \le t_j,\ j = 1,2,\ \ldots,(m-1)$$

Therefore, the holding cost for the j^{th} - replenishment cycle is

$$H_j = \int_{t_{j-1}}^{S_j} h(t)Q_{1j}(t)dt, \quad j = 1,2,\ldots,m \quad (5)$$

The numbers of units deteriorate the during j^{th} - cycle is given by

$$D_j = Q_{1j}(t_{j-1}) - \int_{t_{j-1}}^{S_j}(a+bt)dt, \quad j = 1,2,\ldots,m \quad (6)$$

The ordering cost for j^{th} interval is

$$K_j = K + \beta t_{j-1}, \quad j = 1,2,\ldots,m \quad (7)$$

The total shortages over the j^{th} - replenishment cycle is

$$S_j = \int_{S_j}^{t_j}\left(\int_{S_j}^{t} (a+bu)du\right)dt,\ j = 1,2,\ldots,(m-1) \quad (8)$$

Using (5)-(8), the total cost *TC*, of the inventory system over the planning horizon is

$$TC(m,S_j) = \sum_{j=1}^{m}(K_j + H_j + CD_j) + \pi\sum_{j=1}^{m-1} S_j \quad (9)$$

TC is to be minimized for discrete variable m, the optimal recorders and continuous variable S_j (j = 1,2,… m - 1) the shortage points. For fixed m, the necessary condition for TC to be minimum is

$$\frac{\partial TC}{\partial S_j} = 0, \quad j = 1,2,\ldots,m-1$$

Which means to solve

$$X_1 S_j^{\,2} + X_2 S_j + X_3 = 0, \quad j = 1,2,\ldots,m$$

Where

$$X_1 = e^{-\theta(j-1)\frac{H}{m}}\left((a\theta-b)(h+\alpha(j-1)\frac{H}{m})\frac{3\theta^2}{(-b)}+\frac{\alpha(a\theta-b)}{2\theta}+\frac{3b}{2}(h+\alpha(j-1)\frac{H}{m}+\frac{3b\alpha}{2\theta}\right)$$

$$-\frac{b\alpha}{\theta}+\frac{C\theta(a\theta-b)}{2}-C\theta b-\frac{C\theta b}{2}+b\pi$$

$$X_2 = e^{-\theta(j-1)\frac{H}{m}}\left(\frac{(a\theta-b)(h+\alpha(j-1)\frac{H}{m})}{\theta}+\frac{\alpha(a\theta-b)}{\theta^2}+\frac{2b(h+\alpha(j-1)\frac{H}{m})}{\theta}+\frac{2b\alpha}{\theta^2}\right)$$

$$-\frac{2b\alpha}{\theta^2}-\frac{bh}{\theta}-\frac{a\alpha}{\theta}+\frac{b\alpha}{\theta^2}+\frac{C(a\theta-b)}{\theta}-C(j-1)\frac{H}{m}\theta(a\theta-b)+2bC+\theta bC(j-1)\frac{H}{2m}$$

$$+(j-1)\frac{H}{m}\theta bC+bC-b\pi j\frac{H}{m}+a\pi$$

$$X_3 = e^{-\theta(j-1)\frac{H}{m}}\left(\frac{\alpha(a\theta-b)}{\theta^3}+b(h+\alpha(j-1)\frac{H}{m})+\frac{b\alpha}{\theta^3}\right)-(a\theta-b)\frac{\alpha}{\theta^3}+\frac{\left((a\theta-b)(h+\alpha(j-1)\frac{H}{m}\right)}{\theta^2}$$

$$-\frac{bh}{\theta^2}-\frac{b\alpha}{\theta^3}-\frac{ah}{\theta}+\frac{C(a\theta-b)}{\theta}-\frac{C}{\theta}(a\theta-b)(j-1)\frac{H}{m}+C(a\theta-b)\frac{\theta}{2}(j-1)^2\frac{H^2}{m^2}$$

$$-bC(j-1)\frac{H}{m}-C\frac{\theta b}{2}(j-1)^2\frac{H^2}{m^2}+aC-\pi aj\frac{H}{m}$$

Hence

$$S_j = \frac{-X_2 \pm \sqrt{{X_2}^2-4X_1X_3}}{2X_1}, \quad j=1,2,.......,m \tag{10}$$

4. COMPUTATIONAL ALGORITHM

We find optimal solution numerically using the following algorithm:

Step: 1 $a, b, C, \pi, K, h, \theta, \alpha, \beta, \gamma$ and H. Set m = 2 and let TC (1) = 0.

Step: 2 Set $t_{j-1} = \frac{(j-1)H}{m}$ for $j = 1,2,.......,m$.

Step: 3 For j = 1,2,…., m - 1, find S_j using (10).

Step: 4 Calculate the total cost TC (m) of the inventory system using (9).

Step: 5 If TC (m) < TC (m - 1), then set m = m + 1 and go to step: 2; otherwise go to step: 6.

Step: 6 Set m* = m - 1, $S_j{}^* = S_j\,(m - 1)$, j = 1,2,…., m* - 1 and TC* = TC (m - 1).

Step: 7 Stop.

If $\pi \to \propto$ (i.e. shortages are not allowed) then $S_j = t_j$ for j = 1,2,…,m. Then m* and TC* can be determined easily following the algorithm as stated above.

5. A NUMERICAL EXAMPLE

Consider the parameters of the inventory system as

K = 90	h = 4	a = 10	b = 2	C = 0.5
π = 1.0	θ = 0.1	α = 0.1	β = 0.15	H = 10

The following table shows the effect of changes in various parameters on number of orders to be placed and total cost TC, of an inventory system.

Changes in		m	TC	Changes in		m	TC
a	7	14	151344.91	α	0.09	8	6331.36
	8	12	88994.25		0.10	8	13839.70
	9	10	45582.75		0.20	9	125526.95
	10	8	16839.70		0.30	10	177219.01
	11	6	2531.27		0.40	13	252918.95
b	2	8	16839.70	H	10	8	13839.70
	3	12	39788.47		11	9	97553.80
	4	14	42206.50		12	10	225941.35
	5	16	222789.52		13	11	421955.84
	6	17	284512.31		14	12	703967.12
C	0.5	8	16839.70	h	2.0	12	231804.63
	1.0	8	34910.73		2.5	11	173374.60
	1.5	9	103732.41		3.0	10	112567.07
	2.0	10	218519.89		3.5	9	58916.43
	2.5	11	397378.70		4.0	8	16839.70
π	1	8	16839.70	θ	0.10	8	16839.70
	2	9	35763.55		0.11	9	93780.47
	3	10	58909.00		0.12	10	175065.39
	4	11	88469.17		0.13	11	261086.29
	5	12	136678.19		0.14	12	353041.96

It is observed that

1. An increase in a, reduces the number of orders to be placed and TC of an inventory system.

2. TC and the number of replenishments are very sensitive to changes in b.

3. An increase in π, shortage cost increases the number of orders to be placed over a planning horizon and hence TC, significantly.

4. An increase in α increases the number of orders to be placed over a planning horizon and hence TC, significantly.

5. An increase in planning horizon H, increases the number of replenishment orders and hence TC.

6. An increase in deterioration rate increases the number of orders to be placed during the horizon H.

6. CONCLUDING REMARKS

In the present paper, we have formulated an EOQ model for deteriorating items over a finite time horizon H. The characteristics of the model are.

(i) The deterministic demand is decreasing with time. It is usually observed in the market of electronic components like television, VCD, VCR, freezers, etc.

(ii) There are also shortages.

In many practical situations, stock out is unavoidable due to uncertainties. Also there are situations in which the inventory cost of the stored items is highly compared with its shortage cost.

(iii) Since almost all items undergo either direct spoilage or physical decay in the course of time, deterioration factor has an important role to play in an inventory system.

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