COMPENSATORY LOGIC: A FUZZY NORMATIVE MODEL FOR DECISION MAKING

Rafael Espin Andrade[1], Management Studies Center (CETDIR)
Faculty of Industrial Engineering. ISPJAE (Technical University of Havana), Cuba
Gustavo Mazcorro Téllez, Interdisciplinary College of Engineering, Social and Management Sciences, UPIICSA. México
Eduardo Fernández González, University of Sinaloa, Mexico
Jorge Marx-Gómez, Magdeburg University Otto von Guericke, Alemania
María Ines Lecich, San Juan University, Argentina

ABSTRACT
Fuzzy Logic is a convenient approach for decision making. Its capacity to incorporate verbal statements into formal modeling is advantageous for a systematic treatment of actual situations. This vantage comes from a proper interaction between decision-maker's information and expert reasoning. Many revealing studies deal with multi-valued logics, covering the examination of a variety of operators. However, the real possibility of incorporating "expert knowledge" and the subjectivity inherent to decision makers in practical situations remains limited. This paper proposes an alternative axiomatic multi-valued logic system to overcome such limitation. The system disregards the classical concepts of norm and co-norm. Existential and universal quantifiers are defined consequently, and propositional bivalent classic calculus is also introduced within the logical structure.

Key words: Fuzzy Logic, management, decision making, Multivalued Logic.

MSC: 90B50, 62C99

RESUMEN
La Lógica Difusa ofrece una nueva aproximación, muy conveniente, al problema de la Toma de Decisiones. Su capacidad de transformar expresiones en lenguaje natural a modelos simbólicos, es una ventaja para el tratamiento de problemas reales de decisión, debido a que constituye una manera de modelar tanto desde la perspectiva de las preferencias del decisor, como desde la perspectiva del razonamiento de los expertos. Muchos estudios relevantes tratan sobre Lógicas Multivalentes y acerca de una gama amplia de operadores; sin embargo la posibilidad real de incorporar el conocimiento de los expertos y la subjetividad inherente a los decisores es aún limitada. El presente trabajo propone una alternativa axiomática para definir una nueva lógica multivalente que permita superar esta limitación. El sistema no acude a los clásicos conceptos de norma y conorma, define consecuentemente los cuantificadores existencial y universal, e introduce a partir de esta nueva estructura el clásico cálculo proposicional bivalente.

1. INTRODUCTION

The merge of theoretical knowledge, expert opinions, and subjective information is a convenient demand for rational decision-making; particularly when handling complex and dynamic environments (French 1986, Ostanello 1984). The conditions that arise in modern management, such as breakthroughs in information technology and the prescriptions of innovative managerial paradigms, demand the existence of more flexible analytical tools; some of them devoted to the embodiment of the subtleties of human knowledge and reasoning. When this capacity is enhanced, the organizational intelligence is improved and many strategic goals become more attainable.

The Fuzzy Logic approach has sprouted in early non formalized sciences, such as management. As a result, remarkable progress has been reached in the developing of computing systems for a variety of management tasks, including finances and direction (Von Altrock 1995, Kauffman and Gil Aluj, 1990). The flexibility of Fuzzy Logic admits an effective interpretation of natural language when constructing mathematical models; thus renders more adequate practical conclusions.

Even though the main aspects of Fuzzy Logic are unquestioned, some pragmatic concerns still reveal important levels of perfectibility. One of the well-known applications of Fuzzy Logic is automatic control (Passino

E-mail: [1]espin@ind.ispjae.edu.cu

and Yorkovich 1998). In this regard, it may be said that the use of simple rules as an alternative to the application of intricate procedural patterns has shown better results in some cases. The utilization of average defuzzyfication methods (e.g. Centroid Average and Maximum Center Average methods) (Zimmermann, 1996) attest the convenience of a new logic-system, with compensation as its main feature.

It can be argued, that in the actual practice of managerial decisions, the use of intricate predicates, such as those characterizing expert opinions, discloses the necessity of elaborated subtly frameworks. Hence, in order to cope with the structural complexity of businesses in unstable environments, a better logical approach is required.

On the other hand, the assignment of truth-values to predicates through applications of diverse multi-value logics lack some desirable properties. One of these concerns is the sensibility to changes in truth-values of the basic predicates, or the "verbal meaning" of the truth-values. Likewise, the variations associated with the selection of the connective implication are significant, and it is used to select operators without a logical axiomatic relationship with the previous selected conjunction, disjunction and negation. Therefore, the aim of this paper is to propose a logical system that can cope with some of these drawbacks.

2. BASIC NOTIONS OF FUZZY LOGIC

In Boolean Logic a predicate p is a mapping from the universe set X to {0, 1}. For instance, the sentence '*x is a friend of y*' admits a model in which, according to this logic, the predicate p from the set of pairs (x, y) to {0, 1} assigns 1 if x is effectively a friend of y, and 0 if it cannot be assured that x is a friend of y.

The propositional connectives $\wedge$, $\vee$, y, $\neg$, symbolize operations on sentences.

- $p \wedge q$ is true if and only if both p and q are true. It is called conjunction, and symbolizes the inclusive use of "and" in natural language.
- $p \vee q$ is false if and only if both p and q are false. It is called disjunction, and symbolizes the use of "or" in natural language.
- $\neg p$ is true when p is false, and conversely. It is called the negation of p.

As they are defined in regard to truth values, their functional nature can be addressed as mappings from $\{0, 1\} \times \{0, 1\}$, and ({0;1}, for $\neg p$), to {0;1}.

Thus, for example, if p(*x, y*) symbolizes the sentence '*x is a friend of y*', then the composed sentence '*x is a friend of y, but y is not a friend of x*' should be symbolized by the composed predicate $p(x, y) \wedge \neg p(y, x)$.

The set of predicates built up by the application of $\vee$, $\wedge$, and $\neg$ satisfy a number of properties that render the structure of the classical Boolean Algebra. For any predicate composed by propositional connectives may be assigned a truth function, which specifies truth-values by virtue of the truth-values of the component predicates, in accordance with the truth tables of the propositional connectives.

A fuzzy predicate is a mapping from the universe set X to the continuous interval [0, 1], instead of the classical set {0, 1}.

The introduction of multi-valued logics for modelling practical problems requires category scales, or classifications, which can aid experts to assign truth values or to specify the dependence of predicates on proxy attributes, hence permitting direct assignments. A widely used scale is the following: 0: false; 0.1: nearly false; 0.2: very false; 0.3: somewhat false; 0.4: more false than true; 0.5: as true as false; 0.6: more true than false; 0.7: somewhat true; 0.8: very true; 0.9: nearly true; 1: true.

A multi-valued logic arises from the following definitions:

- $u(p \wedge q) = u(p).u(q)$
- $u(p \vee q) = u(p) + u(q) - u(p).u(q)$
- $u(\neg p) = 1 - u(p)$

in which u(p) is the truth value of p.

The logic so defined does not satisfy transitivity, but satisfies commutativity, associativity, and De Morgan's Laws. This type of logic is usually referred to as probabilistic logic. The definition of connectives is accomplished by the probabilistic expressions for union and intersection of events. However, the assignment does not assume any interpretation in terms of probability.

In addition, the connectives $\wedge$ and $\vee$ satisfy the following properties.

- Any increment in the truth values of p or q increases the truth values of $p \wedge q$, whenever one of the truth values remains at level 0.

- Any increment in the truth values of p, or of q, increases the truth values of $p \vee q$, whenever any of the truth values remains at level 1.

The latter reveals an 'appealing' sensibility, which, in regard to the normative approach to decision analysis, can be related to the axiom of continuity. This property grounds the possibility of modelling veto conditions in decision approaches, as those considered by the so-called French school of decision-aiding. Moreover, it seems convenient for structuring problems concerning the construction of rankings or the selection of alternatives. However, the failure to satisfy the idempotent property causes a loss of verbal significance or its attainment to a category scale; such feature disregards the approach to be used in classificatory or evaluation problems.

Provided the following properties hold:

- $u(p \wedge q) = \min(u(p), u(q))$
- $u(p \vee q) = \max(u(p), u(q))$
- $u(\neg p) = 1 - u(p)$

the most used logic is straightforwardly obtained. It is, of course, the Fuzzy Logic system (e.g. Buckley and Eslami, 2002). This is certainly an extension of the two-value logics; yet it is the unique associative and idempotent system that satisfies properties (1) and (2):

$$\nu(p \wedge q) \le \min(\nu(p), \nu(q)) \tag{1}$$

$$\nu(p \vee q) \ge \max(\nu(p), \nu(q)) \tag{2}$$

Statements (1) and (2) are sufficient to warrant the cardinality of the truth values which is convenient for classification and appraising problems. However, the values of the compound predicates may remain unaffected even under important modifications exerted on the values of basic predicates; thus making the approach unsuitable for ranking or selection problems.

The operators * that satisfy are called average operators.

The definition of implications is rather diverse in Fuzzy Logic (Dubois and Prade 1980, Buckley and Eslami 2002). For example, we may consider the definition $p \to q = \neg p \vee q$, and the so-called Zadeh implication $p \to q = \neg p \vee (p \wedge q)$, which is widely utilized in the theory of control. Similar to Fuzzy Logic (in a narrow sense), the latter satisfies the classical inductive implication '*modus ponens*' in terms of Zadeh's definition of a well-formed deductive structure (Dubois and Prade 1980, Zadeh 1965). Thus, the equivalence is defined by $p \leftrightarrow q = (p \to q) \wedge (q \to p)$ and the universal and existential quantifiers over X are defined by:

$$\forall x\, p(x) = \bigwedge_{x \in X}(p(x))$$

$$\exists x\, p(x) = \bigvee_{x \in X}(p(x))$$

These definitions convey the virtues and defects of the conjunction and disjunction connectives.

3. COMPENSATORY LOGIC

In multivalued logics over [0,1] the conjunction is usually defined as a continuous, associative, and symmetric connective satisfying (1), and the disjunction is defined as an operator implied by De Morgan's Laws according to the definition of conjunction. In such circumstances, the conjunction satisfies the t-norm property, and the disjunction satisfies the t-conorm property.

Notice that properties (1) and (2) lead to the conclusion that the truth-value of the conjunction is equal or less than those of its components; and the truth value of the disjunction is equal or greater than those of its components. The rejection of these properties constitutes the basics of Compensatory Logic (CL). The fundamentals of CL is that an increase or decrease of the truth value of the conjunction or disjunction, as a result of changes in the truth value of one component, can be compensated by an increase or decrease, respectively, of the truth value of other component. This notion yields a very sensible multi-value logic that maintains the categorical values of the truth values. Also, this capacity makes CL especially suited for selection problems; yet it is also convenient for ranking, appraising, and classificatory purposes.

In CL the conjunction *c* is a continuous operator from $[0,1]^n$ to [0,1], so that the following properties hold.

1. $c(1,1,\ldots,1) = 1$

2. If $x_i = 0$ for some i then $c(x_1, x_2, \ldots, x_n) = 0$

3. $c(x_1, x_2, \ldots, x_i, \ldots, x_j, \ldots, x_n) = c(x_1, x_2, \ldots, x_j, \ldots, x_i, \ldots, x_n)$

4. If $x_1 = y_1, x_2 = y_2, \ldots, x_{i-1} = y_{i-1}, x_{i+1} = y_{i+1}, \ldots, x_n = y_n$ are not equal zero, and $x_i > y_i$ then $c(x_1, x_2, \ldots, x_n) > c(y_1, y_2, \ldots, y_n)$

5. $c(x, x, \ldots, x) = x$

In this case, the associativity is excluded because it is incompatible with other desirable properties. Thus, it is necessary to define an associative operator over $[0,1]^n$ as explained below.

Properties (1), (2) guarantee that the restriction of *c* to {0,1} becomes the conjunction in the binary logic system. And property (3) stands for symmetry, but extended to the n-dimensional case.

The idempotent property (5) is a necessary condition for preserving the signification of the truth values, as well as the strict monotony of the operator for all variables, whenever these are not zero. The latter secures the so-called sensibility of the connective predicates without affecting the veto restriction furnished by condition (2). The veto property is fundamental because it introduces the necessary capacity of restriction in compensation when certain goals are not fully satisfied.

From (4), it follows that for all variables the operator *c* is strictly increasing over $[0,1]^n$. This renders the 'sensibility of the conjunctive predicates' regarding changes in basic predicates. Indeed, property (4) determines the impossibility of associativity, which occurs because there are no associative average strictly-increasing operators (Dubois and Prade, 1985), and *c* is an average operator, as proved in what follows.

Let $(x_1, x_2, \ldots, x_n)$ over $[0,1]^n$ be so that $x_m = \min\{x_i\}$ and $x_M = \max\{x_i\}$; then, the strictly increasing property verifies

$$c(x_m, x_m, \ldots, x_m) \le c(x_1, x_2, \ldots, x_n) \le c(x_M, x_M, \ldots, x_M)$$

and, from the idempotent property

$$x_m \le c(x_1, x_2, ..., x_n) \le x_M$$

c is therefore an average operator.

Among the operators discussed in the specialized literature, the unique operator satisfying the previous axiomatic system is the geometric mean. To know,

$$c(x_1, x_2, ..., x_n) = (x_1.x_2.....x_n)^{1/n}$$

Further, the following axioms for negation $n : [0,1] \to [0,1]$ are demanded, whenever these exhibit convenience for the pursued goals:

6. $n(0) = 1$

7. $n(1) = 0$

To introduce a predicate for preference modeling, through the statement "x is better than y", the concept of fuzzy-strict ordering is delineated. Among a variety of reasonable definitions of fuzzy-strict ordering we propose the following:

Let U be the universe set, a predicate o: $U^2 \to [0,1]$ is called a fuzzy-strict ordering if the two conditions below are satisfied:

1. $o(x,y) = n[o(y,x)]$ *(fuzzy reciprocity)*

2. *If* $o(x,y) \ge 0.5$ and $o(y,z) \ge 0.5$ *then* $o(x,z) \ge \max(p(x,y),p(y,z))$ *(fuzzy max-max transitivity)*

In fact, the literature refers this topic in two different ways. Some papers define a strict ordering by assuming fuzzy anti-symmetry and different types of fuzzy transitivity (Dasgupta and Deb 1996, 2001), (Richardson, 1998). But other authors use fuzzy reciprocity and different types of transitivity, yet by holding an approach similar to stochastic treatment of preferences (Chiclana **et al**. 2003), (Garcia-Lapresta and Meneses-Poncio 2001),(García-Lapresta and Marques, 2003), (Switalski 2001, 2003).

Hear is prefered to use fuzzy reciprocity in defining a fuzzy- strict order. Indeed, the fuzzy anti-symmetric property $(p(x,y) > 0 \Rightarrow p(y,x) = 0)$ does not conform to the desired 'sensible performance' related to changes in basic predicates.

Likewise, the choice of the strong fuzzy "max-max" transitivity implies a number of transitivity properties in presence of reciprocity (Garcia-Lapresta and Meneses-Poncio, 2001), (Switalski 2001, 2003), giving more signification to the strict ordering.

The following property is adequate because it renders a natural way to establish a relationship between the measure of convenience and the ordering of alternatives.

8. $o(x, y) = 0.5[C(x) - C(y)] + 0.5$ is a strict ordering over the universe of predicate C.

Property 8 is satisfied if and only if *n(x)=1-x* (see Dubois and Prade 1980 pp. 12-13).

Predicate o(x, y) measures "how much better is x than y" if C measures the convenience of alternatives x and y for a decision- maker. If o(x, y) = 0.5, x and y are indifferent to the decision-maker.

This logic ordering is the same to compare how truer is one or another affirmation when modeled by predicates. It is an ordinal tool to establish a coherent relationship between decision-making properties and deductive properties. Indeed, this approach permits considering decision-making as a deductive-thinking process.

On the other hand, the disjunction must be defined in a sense that satisfies the De Morgan's Laws:

9. $n(c(x_1, x_2,\ldots,x_n) = d(n(x_1), n(x_2),\ldots,n(x_n))$

$n(d(x_1, x_2,\ldots,x_n) = c(n(x_1), n(x_2),\ldots,n(x_n))$

Thus, by taking the geometric means as a definition for conjunction, the disjunction is defined by

$$d(x_1, x_2,\ldots,x_n) = 1 - ((1 - x_1)(1 - x_2)\ldots(1 - x_n))^{1/n}$$

From these properties, similar properties for disjunction can be shown.

This is

10. If $x_i = 1$ for some i then $d(x_1,x_2,\ldots,x_n) = 1$

11. $d(x_1,x_2,x_i,\ldots,x_j,\ldots,x_n) = d(x_1,x_2,x_j,\ldots,x_i,\ldots,x_n)$

12. If $x_1 = y_1$, $x_2 = y_2,\ldots,x_{i-1} = y_{i-1}$, $x_{i+1} = y_{i+1},\ldots,x_n = y_n$ are not

zero, and $x_i > y_i$, then $d(x_1, x_2,\ldots,x_n) > d(y_1, y_2,\ldots,y_n)$

13, $d(x, x,\ldots,x) = x$

Other important properties for decision-making are:

14. $\left| x - c(c(x,y),z)) \right| \geq \left| x - c(x,y,z) \right|$

15. $\left| x - d(d(x,y),z)) \right| \geq \left| x - d(x,y,z) \right|$

These properties of average operators are essential because they establish that the influence of predicates in high levels of a logical definition is more pervasive than those in inferior levels. Notice that these properties contravene associativity, but they are advantageous for decision-making.

The implication can be defined naturally as $i(x,y) = d(n(x),y)$ (Espin, 2002).

For this implication the following properties hold

a) $i(x,y) = 0$ if and only if $x = 1$ and $y = 0$

b) $i(x,y) = 1$ if and only if $x = 0$ or $y = 1$

c) $i(x,y) = i(n(y),n(x))$

d) If $x_1, x_2, y \in \left]0,1\right[$ and $x_1 < x_2$ then $i(x_1,y) > i(x_2,y)$

e) If $x, y_1, y_2 \in \left]0,1\right[$ and $y_1 < y_2$ then $i(x,y_1) > i(x,y_2)$

Such properties guarantee the extensionality of the implication of binary logic, and provide an increasing implication over y, but decreasing over x.

The last characteristic could be useful for modelling deductions, but it renders practical troubles in decision-making, as it will be illustrated latter in this paper.

From properties (a) and (b) it is deduced that the truth values of implication are extended. But property (b) is not recommendable because it restricts the sensibility of the implication when y = 1.

The '*naturality*' of this operator is illusory because the basis of its definition is influenced by the Law of the Excluded middle. Hence it would be better to use $i(x,y) = d(n(x), c(x,y))$, a generalization of the Zadeh implication.

This operator has the following, more satisfactory properties:

16. $i(x,y) = 0$ if and only if $x = 1$ and $y = 0$

17. $i(x,y) = 1$ if and only if $x = 0$ or ($y = 1$ and $x = 1$)

18. If $x, y_1, y_2 \in]0,1]$ and $y_1 < y_2$ then $i(x, y_1) < i(x, y_2)$

19. For $\kappa \in R$, $\kappa \approx 0.4677$, If $y \in]0, \kappa[$ and $x_1 < x_2$ then $i(x_1, y) > i(x_2, y)$

20. For $\kappa \in R$, $\kappa \approx 0.4677$, If $y \in [\kappa, 1]$, it exist $\alpha(y)$

20.1 If $x_1, x_2 \in [0, \alpha(y)]$ and $x_1 < x_2$ then $i(x_1, y) > i(x_2, y)$

20.2 If $x_1, x_2 \in [\alpha(y), 1]$ and $x_1 < x_2$ then $i(x_1, y) < i(x_2, y)$

20.3 $\alpha(y)$ is an strictly decreasing function over $[\kappa, 1]$

Property (16) is the same as property (a) of another implication, and (17) does not restrict sensibility when *y* =1 as it happens in (b). Property (18) assures this sensibility in opposition to (d). Properties (19) and (20) are better than (e) because they take into account the uncertainty, in the same way as conjunction, disjunction, and negation do.

If premises and thesis are 'near' 0.5, the truth values of the implication are located far from 0 or 1, hence they approach closely to 0.5. If both are close to 0 or close to 1, then these values approximate to 0 or 1, and are set far from 0.5.

For the case of limited sets over $R^{n,}$ the existential and universal quantifiers are defined in a natural way from the concepts of disjunction and conjunction, respectively.

$$\forall x\, p(x) = \begin{cases} e^{\frac{\int_X \ln(p(x))dx}{\int_X dx}} & \text{if } p(x) > 0 \text{ for all } x \in X \\ 0 & \text{another case} \end{cases} \tag{4}$$

$$\exists x\, p(x) = \begin{cases} 1 - e^{\frac{\int_X \ln(1-p(x))dx}{\int_X dx}} & \text{if } p(x) > 0 \text{ for all } x \in X \\ 1 & \text{another case} \end{cases} \tag{5}$$

For instance, a composed predicate such as $C(j) = \forall i\,(I_i \rightarrow P_{ji})$ (6), where I_i is the predicate that models the sentence '*the goal i is important*' and P_{ji} expresses the accomplishment of i's aims or desires according to a specified goal by the alternative *j*, is a very general scheme for decision-making. Note that, when predicates associated to attributes are considered, the resulting situation can be identified as the classical multi-criterion decision problem. The quite difficult process of obtaining the weights in constructing normative additive models for decision making (French, 1986) is substituted here for the corresponding truth values of the predicates '*The attribute i is important*' for each attribute.

Still, this approach provides effective inclusion and combination of 'intangibles'-as those obtained by consulting experts and considering categorical scales- with quantitative information obtained by predicates that depend on the involved elements. The importance of the predicates, or attributes, can be measured by a structurally complex process. This has been the case in some models developed by GEMINIS group to solve habitual managerial problems (Espin and Fernández, 1999, Espin, Fernández and Mazcorro, 2002).

To summarize, this approach involves ‘trade offs’ between attributes and the possibility of veto conditions for each attribute; moreover, it permits to model preferential independence by using conditional predicates.

A research project for testing the CL approach in practical situations was carried out. The exploration was accomplished by comparing the application of different multi-valued logics to a variety of managerial decision-making models, all of them based on composed logic predicates (Espin, 2002). The research included problems related to Competitive Positioning, SWOT Matrix Process, Objectives Prioritising, Projects Evaluation, Human Resources Selection by Competences, and Learning Evaluation. For the range of case studies, the multi-valued logics involved were Narrow Fuzzy Logic, Probabilistic Logic, Bounded Logic (it uses the operators bounded sum, and bounded product, such as disjunction and conjunction, see Dubois and Prade, 1980) and CL. The comparisons focused on indexes of sensibility, cardinal value, and appropriateness of results. Fortunately, the best results for all the indexes were those obtained by using CL, hence ratifying the convenience of Compensatory Logic for selection, appraising, ranking, and classification problems.

The formulas of Propositional Compensatory Calculus (PCC) are statements composed by operators c, d, n and i. Following the gist of the introduced Compensatory Predicate Calculus, any mapping $f:[0,1]^n \to [0,1]$ of PCC will be true if f(x) > 0 for any element of the domain, and

$$e^{\frac{\int_{[0,1]^n} \ln(f(x))dx}{\int_{[0,1]^n} dx}} > \frac{1}{2} .$$

In this sense, the satisfied properties are exactly the formulas of Classic Bivalent Propositional Calculus when they use the Natural Implication or the Zadeh Generalized Implication.

The truth values of the formulas pertaining to Kleene’s Axiomatic System were calculated by utilizing the 6.5 version of MATLAB. The involved axioms and the results of the calculus are the following:

$\mathrm{AX1}: \quad A \to (B \to A)$

$\mathrm{AX2}: \quad (A \to B) \to ((A \to (B \to C)) \to (A \to C))$

$\mathrm{AX3}: \quad A \to (B \to A \wedge B)$

$\mathrm{AX4}: \quad A \wedge B \to A \quad A \wedge B \to B$

$\mathrm{AX5}: \quad A \to A \vee B \quad B \to A \vee B$

$\mathrm{AX6}: \quad (A \to C) \to ((B \to C)) \to (A \vee B \to C))$

$\mathrm{AX7}: \quad (A \to B) \to ((A \to \neg B) \to \neg A))$

$\mathrm{AX8}: \quad \neg(\neg A) \to A$

	Natural	Zadeh
Ax 1	0.5859	0.5685
Ax 2	0.5122	0.5073
Ax 3	0.5556	0.5669
Ax 4	0.5859	0.5661
Ax 5	0.8533	0.5859
Ax 6	0.5026	0.5038
Ax 7	0.5315	0.5137
Ax 8	0.5981	0.5981

4. EXAMPLE

A hypothetical case related to multi-attribute decision making is presented below to illustrate the convenience of the Generalized Zadeh Implication in Compensatory Logic.

For this case, four attributes are considered (attributes correspond to rows in the matrixes depicted below). The importance-attribute values, and the column vectors of two alternatives are represented by matrixes $I = [i_j]$ and $P = [p_{kj}]$, respectively.

$$I = \begin{bmatrix} 0.5 \\ 1 \\ 0.5 \\ 0.5 \end{bmatrix} \qquad P = \begin{bmatrix} 1 & 0.8 \\ 0.4 & 0.9 \\ 1 & 0.8 \\ 1. & 0.8 \end{bmatrix}$$

The alternative of the second column of matrix P is obviously better than the alternative in the first column. This arises from the obviously inferior performance of the first alternative in the second attribute, which is also the most important one according to I.

The matrix T_1 of truth values of each implication $i_j \rightarrow P_{kj}$ and the result matrix C_1 using formula 6 yield by natural implication:

$$T_1 = \begin{bmatrix} 1.0000 & 0.6838 \\ 0.2254 & 0.6838 \\ 1.0000 & 0.6838 \\ 1.0000 & 0.6838 \end{bmatrix} \qquad C_1 = \begin{bmatrix} 0.6890 & 0.6838 \end{bmatrix}$$

And the corresponding matrixes by using Generalized Zadeh Implication are**:**

$$T_2 = \begin{bmatrix} 0.6173 & 0.5713 \\ 0.3937 & 0.7735 \\ 0.6173 & 0.5713 \\ 0.6173 & 0.5713 \end{bmatrix} \qquad C_2 = \begin{bmatrix} 0.5517 & 0.6163 \end{bmatrix}$$

Note that first alternative is preferred by the first model, and the second one by the model using Generalized Zadeh Implication. This result reveals an underperformance of the model that uses Natural Implication; the hindrance is a consequence of its incapacity to appreciate adequately the importance of the attributes, since this implication takes the constant value 1 whenever the truth value of consequent is 1, independently of the attribute importance.

5. CONCLUDING REMARKS

Fuzzy Logic models for decision-making are frequently constructed by using aggregation operators. But efficient decision-making needs to be accomplished through a more systematic view. Probabilistic Logic and other multi-valued logics are convenient for selection and ranking problems; and Fuzzy Logic ‘in a narrow sense’ is convenient for appraising, and classification problems. As exposed, a definition of implication may be stated in different ways, hence their convenience depends on the properties of the connectives used.

Compensatory Logic is convenient for selection, ranking, appraising, and classification problems. Aside from the theoretical arguments, the qualities of CL were tested by empirical research in which a variety of Logical Systems were involved to solve managerial problems. The demonstration of traditional Propositional Calculus from this perspective is a good support for the new axiomatic model. Moreover, the functioning of the classical Predicate Calculus system from the new approach was also proved.

REFERENCES

BUCKLEY, J.J. and E. ESLAMI (2002): “An Introduction to Fuzzy Logic and Fuzzy Sets”. **Physica-Verlag Heidelberg**. Germany.

CHICLANA, F., HERRERA, F., HERRERA-VIEDMA, E. and L. MARTINEZ (2003): **“**A note on the reciprocity in the aggregation of fuzzy preference relations using owa operators”. **Fuzzy Sets and Systems**, 137, 71–83.

DASGUPTA, M. and R. DEB (1996): “Transitivity and Fuzzy Preferences”. **Social Choice and Welfare** 13, 305-318.

DASGUPTA, M. and R. DEB (2001): “Factoring Fuzzy Transitivity”. **Fuzzy Set and System** 118, 489-502.

DUBOIS, D. and H. PRADE (1980): **Fuzzy Sets and Systems: Theory and Applications**. Academic Press Inc.

______________________ (1985): “A review of fuzzy set aggregation connectives”. **Information Sciences** 36, 85-121.

ESPIN, R. and E. FERNÁNDEZ. (1999): “Fuzzy Logic Model for Bargaining. Foundation of Computing and Decision Sciences (FCDS)”. **Poznan** 24(4).

ESPIN, R. (2002): **Logical Management Models**. Proceedings of AMSE Conference. Girona, Spain.

ESPIN, R.; E. FERNÁNDEZ and G. MAZCORRO (2002): Compensatory Logic. Proceedings of the Congress of Fuzzy Logic for Management and Economy. **SIGEF 2002**. Mérida. Venezuela.

FRENCH, S. (1986): **Decision Theory: An Introduction to the Mathematics of Rationality**. NY-Brisbane –Toronto. Halsted Press.

GARCÍA-LAPRESTA, J.L. and L.C. MENESES-PONCIO (2001): “An empirical analysis of fuzzy transitivities in Decision Making”***,*** **VIII SIGEF Congress Proceedings**, 129-133.

GARCÍA-LAPRESTA, J.L. and R.A. MARQUES (2003): “Constructing reciprocal and stable aggregation operators”. **Proceedings AGOP 2003**, Alcala de Henares, 73-78.

GIL-ALUJA, J. (1999): **Elementos para una teoría de la Decisión en la Incertidumbre**. Editorial Milladoirro. Barcelona. España.

KAUFMANN A.Y. and J. GIL ALUJA (1990): **Las matemáticas del azar y de la incertid**umbre. Editorial Centro de estudios Ramón Areces, Madrid.

OSTANELLO, D. (1984): ”Outranking Relations”. Summer School of MCDM.Sicilia.

PASSINO, K. and S. YORKOVICH (1998): **Fuzzy Control**. Addison Wesley. New York.

SWITALSKI, Z. (2001): “Transitivity of Fuzzy Preferences relations: An empirical study”. **Fuzzy Sets and Systems** 118 503-508.

SWITALSKI, Z. (2003): General Transitivity conditions for fuzzy reciprocal preference matrices, **Fuzzy Sets and Systems** 137 (1) 85-100.

VON ALTROCK, C. (1995): **Fuzzy Logic and Neurofuzzy Applications in Bussines and Fina nce**. Prentice Hall, New Jersey.

ZADEH, L. A. (1965): Fuzzy Sets**. Inf. Control** 8, 338-353.

ZIMMERMANN, H.J (1996): **Fuzzy Set Theory and its applications,** Kluwer Academic Publishers, Boston-Dordrecht-London.