AN EOQ MODEL FOR TWO LEVEL CREDIT POLICY ORDER RANDOM INPUT

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ABSTRACT:

In this article, a mathematical model is developed to study the optimal retailer's replenishment decisions under two levels trade credit scenario when production rate of the supplier is finite and units received by retailer are random. By two levels of trade credit means that the supplier offer a delay period, M (say) to the retailer and retailer passes it (say) N, to his customer. The retailer's credit period N offered to the customer is less than the supplier's allowable credit period M to the retailer. The optimal replenishment policy is established by minimizing total expected cost of an inventory system. The easy-to-use algorithm is given to search for optimal replenishment policy. Finally, numerical examples are given to validate the derived model.

Key Words: EPQ, inventory, two level trade credit policy, random input.

RESUMEN

En este trabajo se desarrolla un modelo matemático para estudiar las decisiones de reabastecimiento óptimas del vendedor bajo dos niveles de escenarios de créditos cuando la tasa de producción del suministrador es finita y las unidades recibidas por el vendedor son aleatorias. Se entiende por dos niveles de crédito que el suministrador ofrece un periodo espera, (digamos M) para el vendedor y este le despacha este (digamos N) al comprador. El periodo de crédito que el vendedor ofrece al comprador es menor que el ofertado por el permitido por el suministrador M al vendedor. La política optima de reabastecimiento es fijada para minimizar el consto total esperado de un sistema de inventario. El algoritmo de facial uso es dado para buscar la política de reabastecimiento optima. Finamente se dan ejemplos numéricos para validar el modelo derivado.

1. INTRODUCTION

Harris and Wilson's EOQ model is based on the two stringent assumptions, namely,

i) The retailer must pay for the units as soon as the items are received in his system, and
ii) The items received by the retailer exactly matches with order requisitioned.

However, this may not be true. In practice, the supplier offers credit period to the retailer for settling the amount of purchasing cost. Before the end of the allowable delay period, the retailer can sell the units and accumulate revenue and earn interest. A higher interest is charged if the payment is not settled by the end of the credit period. Thus, retailer is tempted to delay his payment upto the last moment of the credit period given by the supplier. In fact, now-a -days, supplier considers this to be promotional policy for his commodities and also to attract more retailers.

Secondly, due to variety of reasons, viz, machine's breakdown, worker's strike, electricity failure, shortage of raw materials etc., it is found that quantity received does not necessarily match with the quantity ordered but may be a random variable depending on the quantity ordered. Silver (1976) developed an EOQ model when the quantity received is uncertain. Kalro and Gohil (1982) extended Silver's result to allow shortages. Shah and Shah (1992-a, 1992-b) developed optimal ordering policy when units received are random in nature and subject to constant rate of deterioration for infinite/finite production rate. Gor and shah (1994) extended above said model by allowing shortages. Yano and Lee (1995) reviewed the literature on quantitatively oriented approach for determining lot-sizes when production or procurement yields are random. They discussed the issues relating to the modeling cost yield uncertainty and performance of the system with random yield.

Goyal (1985) established a single-item inventory model when supplier offers delay period to settle the account of the retailer. Chand and Ward (1987) analyzed Goyal's model (1985) and gave different result. Chung (1998-a, 1998-b) derived an alternative method to determine the economic order quantity under condition of permissible delay period. Shah (1993), Aggarwal and Jaggi (1995), Chu et al (1998), Chu et al (2001) derived lot-size model when units in an inventory deteriorate at a constant rate under the conditions of permissible delay in payments. Chung (2000) developed an alternative approach to modify Shah's (1993) solution. Chung et al (2001) derived model by assuming demand to be linear function of time. Chen and chuang (1999), shah et al (2004) derived model with stock dependent demand under trade credit approach. Related articles are by Kim et al (1995), Hwang and shinn (1997), Jamal et al (2000), Sarker et al (2001), Khouja and Mehrez (1996), Shwaky and Abou-El-Ata (2001), Teng (2002), shinn and Hwang (2003), Arcelus et al (2003) etc.

The above models are derived under the assumptions that the supplier offers the retailer a trade credit period and could sell the units and earn interest within the allowable credit period. It is implicitly assumed that the customer would pay for the units at the time of purchase. Here, an attempt is made to derive a model when the retailer also passes some credit period to stimulate his customer demand. When units received by the retailer are random, it is assumed that the retailer's credit period, M, offered by the supplier is greater than that of customer, offered by retailer say N, (M. ≥ N). The situation M < N is unrealistic because retailer cannot earn interest. The replenishment rate is assumed to be finite. This is known as economic production quantity (EPQ) model. Under these conditions, retailer's inventory system is modeled to derive optimal retailer's replenishment decisions under two levels trade credit policy within the EPQ environment when units received by the retailer may not exactly match with that of requisitioned. An easy-to-use algorithm is given to determine the optimal replenishment decisions for the retailer. The numerical examples are given to supply the modeling.

2. ASSUMPTIONS AND NOTATIONS

The model is derived under the following assumptions:

- The demand rate, R, is known and constant.
- The replenishment rate, P, is finite, known and constant. (P ≥ R).
- Lead-time is zero. Shortages are not allowed.
- The planning horizon is infinite.
- The order size is Q-units per replenishment, however, actual quantity received is Y, follows normal distribution with

$$E(y) = bQ \qquad (2.1)$$

$$V(y) = \sigma_0^2 + \sigma_1^2 Q^2$$

Where b > 0 is the bias factor, σ_0^2 and σ_1^2 are known constants.

When T, (a cycle time when Q units are ordered) ≥ M, the account is to be settled at T = M and the retailer pays interest charges on the unsold items with rate I_c, when T ≤ M, the account is settled at T = M and the retailer does not have to pay any interest charges. The retailer can deposit his revenue in an interest bearing account at the rate of Ie/unit/year i.e. retailer can earn interest during interval [N, M] with rate Ie under permissible delay period.

Notations:

- R = demand/ annum
- P = replenishment rate/ year, (P ≥ R)
- A = ordering cost/ order
- $\rho = 1 - \frac{R}{P} > 0$
- C = unit purchase cost/ unit
- h = unit inventory holding cost/ unit/ year excluding interest charges
- I_e = interest earned/ $/ year
- I_c = interest charges/ $ in stock/ year by the supplier
- M = the retailer's permissible credit period offered by supplier in years.
- N = the customer's trade credit period offered by retailer in years, M ≥ N.
- Q (t / Y) = on hand inventory of the system at time t of a cycle when actually y-units are received.
- T (Y) = $\frac{Y}{R}$, cycle time when y units are received.
- Q = the procurement quantity to be ordered (a decision variable)
- TEC (Q) = the total expected cost of the system when Q units are ordered.
- Q* = the optimal procurement quantity.
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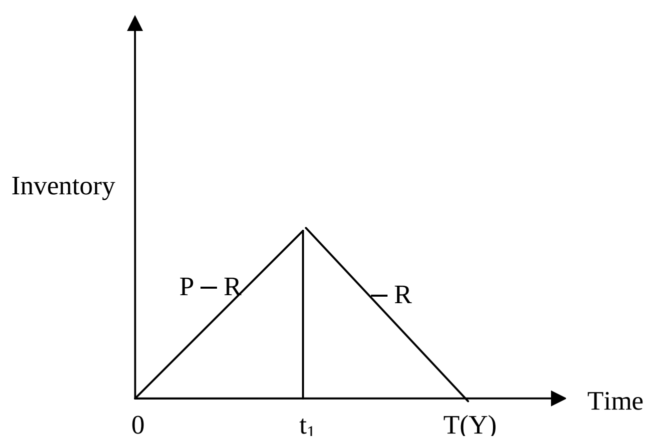
3. MATHEMATICAL FORMULATION



The inventory cycle consists of two phases. During the phase – I, period (0, t1), units are produced at a constant rate P and demand occurs at a constant rate of R-units, leaving a balance of (P – R) units to enter into the inventory system. During phase – II, period (t_1, T(Y)), there is no production and demand occurring at a constant rate R is meet from the inventory accumulated during phase – I period. Let Q (t/Y) denotes on-hand inventory of the system at time t of a cycle when actually Y-units are received by the system. Then the differential equation that governs the instantaneous states of Q (t/Y) is given by

$$\frac{dQ(t/Y)}{dt} = \begin{cases} P-R, & 0 \le t \le t_1 \\ -R, & t_1 \le t \le T(Y) \end{cases} \tag{3.1}$$

with initial condition Q (t/Y) = 0 = Q (T(Y)/Y).
Then solution of differential equation (3.1) is

$$Q\,(t/Y) = \begin{cases} (P-R)t, & 0 \le t \le t_1 \\ R(T(Y)-t), & t_1 \le t \le T(Y) \end{cases} \tag{3.2}$$

Q (t/Y), being a continuous function of t, at t = t_1 we have (P – R) t_1 = R (T(Y) – t_1). So $t_1 = \frac{Y}{P}$. Hence, inventory, I_1(Y), during [0, T(Y)] in the system is

$$I_1(Y) = \int_0^{T(Y)} Q(t/Y)dt = \int_0^{t_1} (P-R)t\ dt + \int_{t_1}^{T(Y)} R(T(Y)-t)dt = \frac{Y^2}{2R}\rho$$

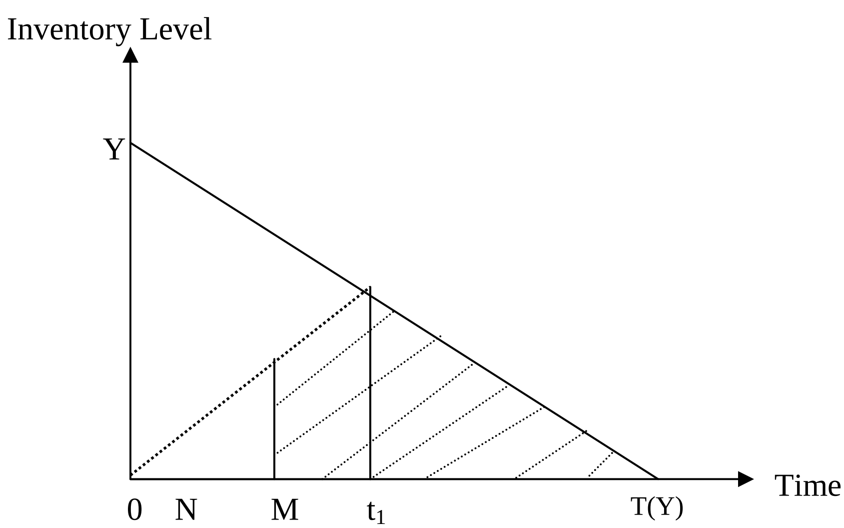
Therefore, annual inventory holding cost excluding interest charges, IHC (Y), for the cycle is IHC (Y)

$$= \frac{hY^2}{2R}\rho \tag{3.3}$$

Annual ordering cost; OC; for the cycle is OC = A (3.4)

The other two components of total cost of an inventory system, viz. interest charged and interest earned are discussed in different situations as follows:

Case 1: $M \le t_1 \le T(Y)$



Interest earned, IE_1, per cycle is

$$IE_1 = CI_e \int_N^M Rt\ dt = \frac{CI_e R}{2}(M^2 - N^2) \tag{3.5}$$

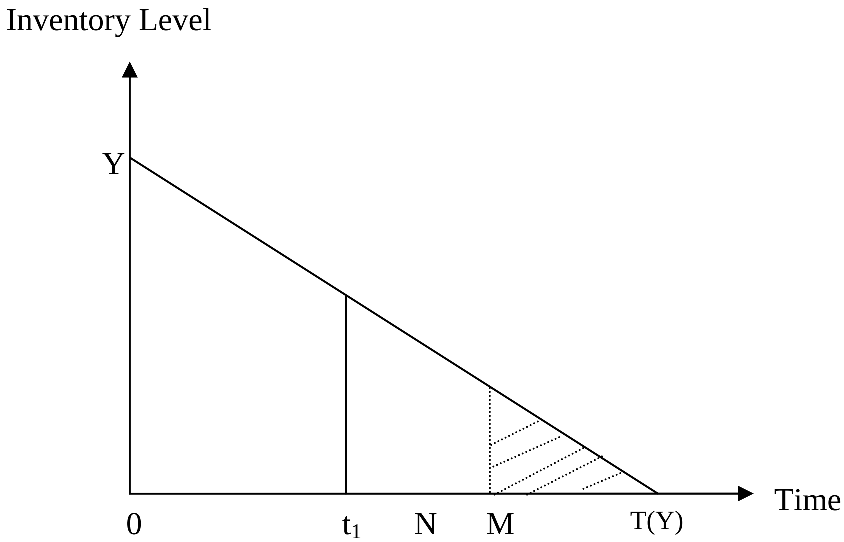
and interest charged, IC_1, per cycle is

$$IC_1 = CI_c \int_M^{t_1} Q(t)dt = \frac{CI_c(P-R)}{2}\left(\frac{Y^2}{P^2} - M^2\right) \tag{3.6}$$

Using (3.3)-(3.6), total cost, TC_1(Y), of an inventory system when Y-units are received is

$TC_1(Y) = OC + IHC + IC_1 - IE_1$ (3.7)

Case 2: $t_1 \le M \le T(Y)$

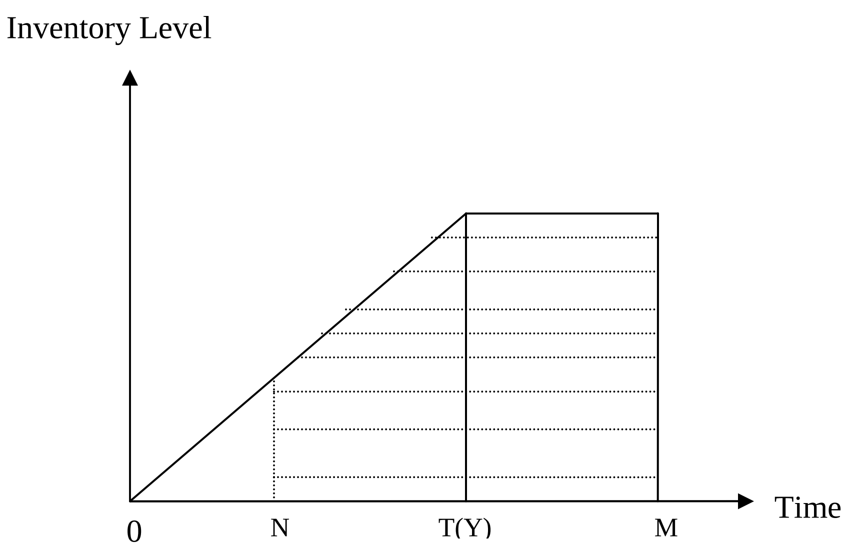


Interest earned, IE_2, is same as IE_1 and interest charged, IC_2 over the cycle is

$$IC_2 = CI_c \int_M^{T(Y)} R(T(Y)-t)dt = \frac{CI_c}{2}\left(Y-RM\right)^2 \quad (3.8)$$

Arguing as in case-1, total cost, $TC_2(Y)$, of an inventory system when Y-units are received is
$TC_2(Y) = OC + IHC + IC_2 - IE_2$ (3.9)

Case 3: $N \le T(Y) \le M$



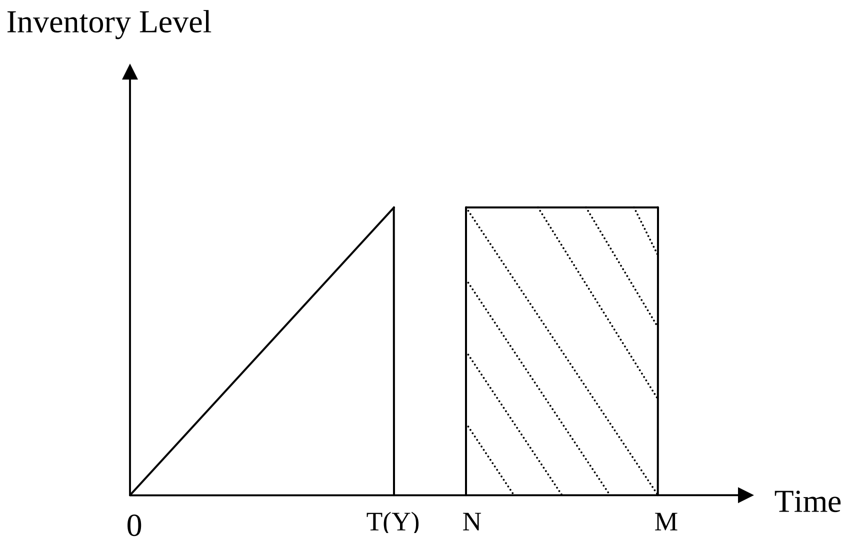
In this case, interest charged, $IC_3 = 0$ and interest earned, IE_3 is

$$IE_1 = CI_e \left\{ \int_N^{T(Y)} Rtdt + RT(Y)(M-T(Y)) \right\} = \frac{CI_e}{2R}\left\{-Y^2 - R^2N^2 + 2\ Y\ M\ R\right\} \quad (3.10)$$

Hence, total cost, $TC_3(Y)$, of an inventory system when Y-units are received is

$TC_3(Y) = OC + IHC + IC_3 - IE_3$ (3.11)

Case 4: $T(Y) \le N \le M$



Here, also interest charged, $IC_4 = 0$. Interest earned, IE_4 over the cycle is

$$IE_4 = C\ I_e\ R\ T(Y)\ (M-N) = C\ I_e\ Y(M-N) \qquad (3.12)$$

The total cost, $TC_4(Y)$, of an inventory system when Y-units are received is

$TC_4(Y) = OC + IHC + IC_4 - IE_4$ (3.13)

Using (2.1), total expected cost when Q-units are ordered per time unit is given by

$$TEC_1(Q) = \frac{AR}{bQ} + \frac{h\rho}{2bQ}(\sigma_0^2 + (\sigma_1^2 + b^2)Q^2) + \frac{C\ I_c(P-R)R}{2bQ}\left(\frac{\sigma_0^2 + (\sigma_1^2 + b^2)Q^2}{P^2} - M^2\right)$$
$$-\frac{CI_eR^2(M^2-N^2)}{2bQ} \qquad M \le t_1 \le T \qquad (3.14)$$

$$TEC_2(Q) = \frac{AR}{bQ} + \frac{h\rho}{2bQ}(\sigma_0^2 + (\sigma_1^2 + b^2)Q^2) + \frac{C\ I_cR}{2bQ}\left(\sigma_0^2 + (\sigma_1^2 + b^2)Q^2 - 2bQMR + R^2M^2\right)$$
$$-\frac{CI_eR^2(M^2-N^2)}{2bQ} \qquad t_1 \le M \le T \qquad (3.15)$$

$$TEC_3(Q) = \frac{AR}{bQ} + \frac{h\rho}{2bQ}(\sigma_0^2 + (\sigma_1^2 + b^2)Q^2) - \frac{C\ I_e}{2bQ}\left(-\sigma_0^2 - (\sigma_1^2 + b^2)Q^2 - R^2N^2 + 2bQMR\right)$$
$$N \le T \le M \qquad (3.16)$$

$$TEC_4(Q) = \frac{AR}{bQ} + \frac{h\rho}{2bQ}(\sigma_0^2 + (\sigma_1^2 + b^2)Q^2) - C\ I_eR\ (M-N) \qquad T \le N \le M \qquad (3.17)$$

All TEC_i (Q), i = 1, 2, 3, 4 are continuous and differentiable. Equations (3.14) – (3.17) give

$$TEC_1'(Q)=\left(\frac{h(\sigma_1^2+b^2)Q\rho}{R}+\frac{CI_c(P-R)(\sigma_1^2+b^2)Q}{P^2}\right)\frac{R}{bQ}-\frac{1}{bQ^2}\Big\{(AR+\frac{h\rho}{2b}(\sigma_0^2+(\sigma_1^2+b^2)Q^2)$$

$$+\frac{C\ I_cR}{2}\left(\sigma_0^2+(\sigma_1^2+b^2)Q^2-M^2\right)-\frac{CI_eR(M^2-N^2)}{2}\Big\} \tag{3.18}$$

$$TEC_2'(Q)=\left(\frac{h(\sigma_1^2+b^2)Q\rho}{R}+CI_c(\sigma_1^2+b^2-2bMR)\right)\frac{R}{bQ}-\frac{1}{bQ^2}\Big\{(AR+\frac{h\rho}{2b}(\sigma_0^2+(\sigma_1^2+b^2)Q^2)$$

$$+\frac{C\ I_cR}{2}\left(\sigma_0^2+(\sigma_1^2+b^2)Q^2-2bQMR+R^2M^2\right)-\frac{CI_eR^2(M^2-N^2)}{2}\Big\} \tag{3.19}$$

$$TEC_3'(Q)=\left(h(\sigma_1^2+b^2)Q\rho+CI_e(\sigma_1^2+b^2-2bMR)\right)\frac{1}{bQ}-\frac{1}{bQ^2}\Big\{(AR+\frac{h\rho}{2b}(\sigma_0^2+(\sigma_1^2+b^2)Q^2)$$

$$+\frac{C\ I_e}{2}\left(\sigma_0^2+(\sigma_1^2+b^2)Q^2-2bQMR+R^2N^2\right)\Big\} \tag{3.20}$$

$$TEC_4'(Q)=\left(h(\sigma_1^2+b^2)Q\rho-CI_ebR(M-N)\right)\frac{1}{bQ}-\frac{1}{bQ^2}\Big\{(AR+\frac{h\rho}{2}(\sigma_0^2+(\sigma_1^2+b^2)Q^2)$$

$$-C\ I_eRbQ(M-N)\Big\} \tag{3.21}$$

$$TEC_1''(Q)=\frac{1}{P^2bQ^3}\Big\{2ARP^2+h\rho P^2\sigma_0^2-CI_cR^2+CI_cRP\sigma_0^2+CI_cR^2M^2P^2-CI_cRM^2P^3$$

$$+CI_eR^2N^2P^2-CI_eR^2M^2P^2\Big\} \tag{3.22}$$

$$TEC_2''(Q)=\left(\frac{h(\sigma_1^2+b^2)\rho}{R}+CI_c(\sigma_1^2+b^2)\right)\frac{R}{bQ}-\frac{2R}{bQ^2}\frac{h(\sigma_1^2+b^2)Q\rho}{R}$$

$$+CI_c(\sigma_1^2+b^2-2bMR)-\frac{2R}{bQ^2}\Big\{(A+\frac{h\rho}{2R}(\sigma_0^2+(\sigma_1^2+b^2)Q^2)$$

$$+\frac{C\ I_c}{2}\left(\sigma_0^2+(\sigma_1^2+b^2)Q^2-2bQMR+R^2M^2\right)-\frac{CI_eR^2(M^2-N^2)}{2}\Big\} \tag{3.23}$$

$$TEC_3''(Q)=\frac{1}{bQ}\left(h(\sigma_1^2+b^2)\rho+\frac{CI_e(\sigma_1^2+b^2)}{2}\right)-\frac{2}{bQ^2}h(\sigma_1^2+b^2)Q\rho$$

$$+\frac{CI_e(2(\sigma_1^2+b^2)Q-2bMR)}{2}-\frac{2R}{bQ^3}\Big\{(A+\frac{h\rho}{2R}(\sigma_0^2+(\sigma_1^2+b^2)Q^2)$$

$$+\frac{C\ I_e}{2R}\left(\sigma_0^2+(\sigma_1^2+b^2)Q^2-2bQMR+R^2N^2\right)\Big\} \tag{3.24}$$

$$TEC_4''(Q)=\frac{h(\sigma_1^2+b^2)\rho}{bQ}-\frac{2R}{bQ^2}\left(\frac{h(\sigma_1^2+b^2)Q\rho}{R}+CI_eb(M-N)\right)$$

$$-\frac{2R}{bQ^3}\Big\{(A+\frac{h\rho}{2R}(\sigma_0^2+(\sigma_1^2+b^2)Q^2)+C\ I_ebQ(M-N)\Big\} \tag{3.25}$$

Discussions:

Each of the TEC $_i$ (Q), i = 1, 2, 3, 4 are convex. TEC $_i$ (Q) is convex iff

$$\alpha = 2ARP^2 + h\rho P^2\sigma_0^2 - CI_cR^2\sigma_0^2 + CI_eRP\sigma_0^2 + CI_cR^2M^2P^2 - CI_cRM^2P^3$$
$$+ CI_eR^2P^2(N^2 - M^2) > 0$$

Necessary condition: Equating $TEC_i'(Q) = 0$, i = 1, 2, 3, 4, we get

$$Q_1^* = \sqrt{\frac{2ARP^2 + h\rho P^2\sigma_0^2 + CI_cR\sigma_0^2(P-R) - CI_cRM^2P^2(P-R) + CI_eR^2P^2(N^2-M^2)}{\left(h\rho P^2 + CI_cR(P-R)\right)\left(\sigma_1^2 + b^2\right)}} \quad (3.26)$$

$$Q_2^* = \sqrt{\frac{2\,A\,R + h\,\rho\,\sigma_0^2 + C\,I_c\,R\,\sigma_0^2 + C\,I_c\,R^3M^2 + C\,I_e\,R^2(N^2-M^2)}{\left(h\rho + CI_cR\right)\left(\sigma_1^2 + b^2\right)}} \quad (3.27)$$

$$Q_3^* = \sqrt{\frac{2\,A\,R + (h\,\rho + C\,I_c)\,\sigma_0^2 + C\,I_e\,R^2N^2}{\left(h\rho + CI_e\right)\left(\sigma_1^2 + b^2\right)}} \quad (3.28)$$

$$Q_4^* = \sqrt{\frac{2\,A\,R + h\,\rho\,\sigma_0^2}{h\rho\left(\sigma_1^2 + b^2\right)}} \quad (3.29)$$

Also,

$$TEC_1'\left(\frac{PM}{b}\right) = TEC_2'\left(\frac{PM}{b}\right) = \frac{-1}{2bP^3M^2}\Big\{-h\rho P^3[M^2(\sigma_1^2 + 2b^2) + \frac{\sigma_0^2b^2}{P}] - CI_cRP^2M^2(\sigma_1^2 + b^2)$$
$$+ CI_cR^2PM^2(\sigma_1^2 + b^2) + CI_cR\sigma_0^2b^2(P-R) - CI_cRM^2P^2b^2(P-R) - CI_eR^2P^2b^2(M^2 - N^2)\Big\}$$
(3.30)

$$TEC_2'\left(RM\right) = TEC_3'\left(RM\right) = \frac{1}{2bR^2M^2}\Big\{h\rho[R^2M^2(\sigma_1^2 + b^2) - \sigma_0^2] - CI_cR^3M^2(\sigma_1^2 + b^2)$$
$$-2AR - CI_cRM^2(\sigma_0^2 + R^2M^2) + CI_eR^2(M^2 - N^2)\Big\}$$
. (3 .31)

$$TEC_3'\left(RN\right) = TEC_4'\left(RN\right) = \frac{1}{2bR^2N^2}\Big\{h\rho[R^2M^2(\sigma_1^2 + b^2) - \sigma_0^2] - CI_eR^2M^2(\sigma_1^2 + b^2)$$
$$-2AR - CI_eRM^2(\sigma_0^2 + R^2N^2)\Big\}$$
. (3.32)

Define

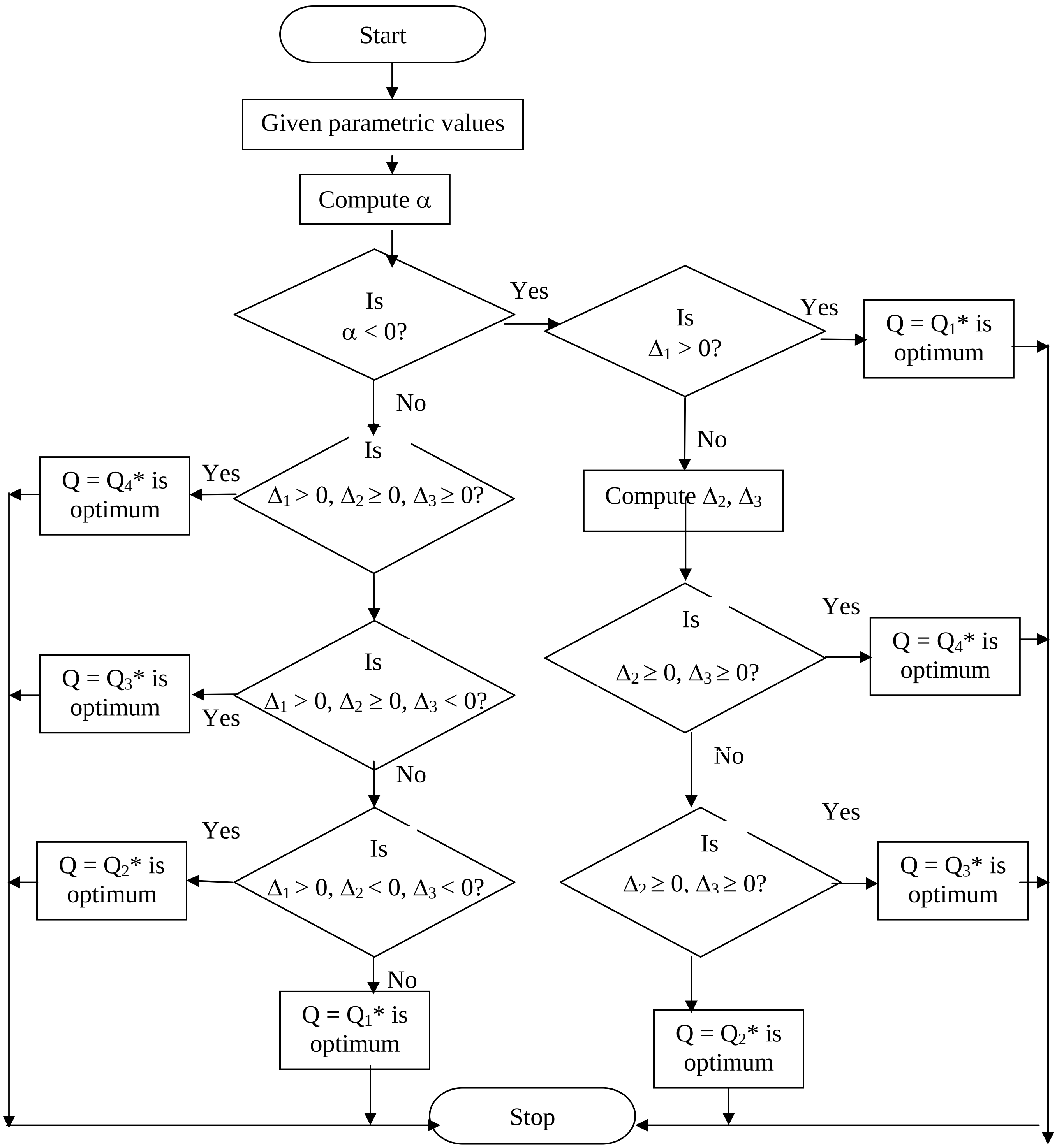
$$\Delta_1 = h\rho P^2(P^2M^2(\sigma_1^2 + b^2) + \sigma_0^2b^2) + CI_cRP^2M^2(\sigma_1^2 + b^2) - CI_cR^2PM^2(\sigma_1^2 + b^2) - 2Ab^2RP^2$$
$$CI_cRPb^2(\sigma_0^2 + MP^2) + CI_cR^2b^2(\sigma_0^2 - M^2P^2) + CI_eR^2P^2b^2(M^2 - N^2)$$
. (3.33)

$$\Delta_2 = h\rho(R^2M^2(\sigma_1^2 + b^2) - \sigma_0^2) + CI_cR^3M^2(\sigma_1^2 + b^2) - 2AR - CI_cR(\sigma_0^2 + R^2M^2)$$
$$+ CI_eR^2(M^2 - N^2) \quad (3.34)$$

$$\Delta_3 = h\rho(R^2N^2(\sigma_1^2 + b^2) - \sigma_0^2) + CI_eP^2N^2(\sigma_1^2 + b^2) - 2AR - CI_e(\sigma_0^2 + R^2N^2) \quad (3.35)$$

In the next section, we exhibit computational algorithm.

4. COMPUTATIONAL ALGORITHM:



5. SPECIAL CASE

When $P \to \infty$, M = N = 0, derived model reduces to that of Silver (1976).

When $P \to \infty$, N = 0, $\sigma_0^2 = \sigma_1^2 = 0$ and b = 1, the developed model is same as that of Goyal (1985).

6. NUMERICAL EXAMPLES

Data is defined in proper units.

Example 6.1: Consider M = 0.1, N = 0.08, I_C = 15%, Ie = 12%, A = 30, R = 5000, P = 9000, C = 100, h = 5, $\sigma_0^2 = 5$, $\sigma_1^2 = 0.1$, b = 0.75.

Then $\alpha < 0,\ \Delta_2 < 0,\ \Delta_3 < 0$. Using algorithm, $Q = Q_2^*$ =614.28 minimizes total expected cost $TEC_2(Q_2^*)$ = 3197214.86.

Example 6.2: Consider M = 0.1, N = 0.08, I_C = 15%, Ie = 12%, A = 100, R = 3000, P = 6000, C = 100, h = 5, $\sigma_0^2 = 5$, $\sigma_1^2 = 0.1$, b = 0.95.

Then $\alpha < 0,\ \Delta_2 > 0,\ \Delta_3 < 0$. Using algorithm, $Q = Q_3^*$ =298.05 is optimum units to be procured and corresponding minimum cost is 960.49.

Example 6.3: Consider M = 0.1, N = 0.08, I_C = 15%, Ie = 12%, A = 100, R = 2000, P = 7000, C = 100, h = 5, $\sigma_0^2 = 5$, $\sigma_1^2 = 0.1$, b = 0.90.

Then $\alpha < 0,\ \Delta_2 < 0,\ \Delta_3 < 0$. Hence optimum $Q = Q_2^*$ =209.68 and corresponding minimum cost is 360972.76.

Example 6.4: Consider M = 0.1, N = 0.08, I_C = 15%, Ie = 12%, A = 35, R = 1200, P = 2000, C = 40, h = 25, $\sigma_0^2 = 5$, $\sigma_1^2 = 0.1$, b = 0.95.

Then $\alpha > 0,\ \Delta_2 > 0,\ \Delta_3 > 0$. $Q = Q_4^*$ =91.29 units and minimum cost is $TEC_4(Q_4^*)$ =851.06.

Example 6.5: Consider M = 0.1, N = 0.08, I_C = 15%, Ie = 12%, A = 60, R = 4800, P = 5000, C = 50, h = 10, $\sigma_0^2 = 5$, $\sigma_1^2 = 0.1$, b = 0.95.

Then $\alpha > 0,\ \Delta_1 > 0,\ \Delta_2 > 0, \text{and } \Delta_3 < 0$. Hence, optimum procurement quantity is $Q = Q_3^*$ =477.15 and minimum cost is $TEC_3(Q_3^*)$ = 342.55.

Example 6.6: Consider M = 0.1, N = 0.08, I_C = 15%, Ie = 12%, A = 200, R = 2000, P = 4000, C = 100, h = 5, $\sigma_0^2 = 5$, $\sigma_1^2 = 0.1$, b = 0.90.

Then $\alpha > 0,\ \Delta_1 < 0,\ \Delta_2 < 0, \text{and } \Delta_3 < 0$. Algorithm suggests that $Q = Q_2^*$ = 209.72 and the corresponding minimum cost is 361918.89.

Example 6.7: Consider M = 0.1, N = 0.08, I_C = 15%, Ie = 12%, A = 200, R = 1500, P = 2000, C = 100, h = 5, $\sigma_0^2 = 5$, $\sigma_1^2 = 0.1$, b = 0.90.

Then $\alpha > 0,\ \Delta_1 < 0,\ \Delta_2 < 0, \text{and } \Delta_3 < 0$. Using algorithm $Q = Q_1^*$ = 324.93 is optimal procurement quantity and optimal total expected cost of inventory system is 1334.71.

7. CONCLUSIONS:

The retailer's replenishment policy is studied here by passing some credit period to his customer and hence attracting more customers, as a result, retailer increases his sale. The model assumes that the replenishment rate is finite and order received does not match with order requisitioned, but a random variable following normal distribution. The model can be extended to incorporate more realistic situation like allowing shortage, deterioration of units in an inventory system, demand to be price dependent, time dependent etc.

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