A NOTE ON MARKETING POLICIES: THE CASE OF EXPONENTIALLY DETERIORATING INVENTORY WITH VARIABLE MARKUP UNDER RANDOM INPUT

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RESUMEN
Se ofrece un modelo del inventario se desarrolla bajo diversas políticas de comercialización usando el concepto del margen de beneficio variable cuando las unidades en inventario están conforme a la deterioración, los descuentos de cantidad y la cantidad recibida es una variable al azar. El efecto de la deterioración de unidades y de los varios parámetros en cantidad óptima del margen de beneficio y de la consecución se estudia. El modelo derivado se apoya con un ejemplo numérico

ABSTRACT:
An inventory model is developed under different marketing policies using the concept of variable markup when units in inventory are subject to deterioration, quantity discounts are offered and quantity received is a random variable. The effect of deterioration of units and various parameters on optimum markup and procurement quantity is studied. The model derived is supported with a numerical example.

Key-Words : Marketing policies, deterioration of units, price dependent deterministic demand.

MSC 90B05

1 INTRODUCTION :

Arcelus and Srinivasan (1987) extended the classical EOQ model to reflect various optimizing criteria and alternative demand and price structure in order to develop decision rule for the management of finished goods inventories, specially for retailing, in which inventories are evaluated in the same way as any other investments, such as the maximization of generating profit or return on investment rather than the traditional cost minimization approach. This is in contrast to the usual least cost objective of raw material or work – in – process inventory management, where inventory costs are viewed as another input cost of production. Also in traditional EOQ model, it is implicitly assumed that the quantity received per order matches with the quantity received, however in practice, it happens that due to variety of reasons, the quantity received differs from that of ordered. Silver (1976) developed an EOQ model when quantity received is uncertain and is a random variable with some specified mean and variance. In such a case, the best ordered quantity depends on the mean and standard deviation of the amount received.

This paper studies in detail the analytical implementation of different cost structures and their effect on inventory policies when the quantity received per order does not match with the quantity requisitioned. The objective is the maximization of average expected profit (ANP) and average expected residual income (ARI). For further details on the use of these two policies refer Sankarsubramanyam and Kumarswamy (1981), Morse and Schneider (1979), Schroder and

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Krishna (1976), Ladany and Sternlib (1974), Kotler (1971), Agarwal and Ayalawadi (1988), Jani (1978), Brahambhatt (1981) etc..

2. ASSUMPTIONS :

The model is developed with following assumptions :

1. The unit cost C(Q) is a function of the order quantity and it satisfies $\frac{\partial C(Q)}{\partial Q} \le 0$. Following Subramanian and Kumarswamy (1981), C(Q) is assumed to have the form

$$C(Q) = h_1 - h_2 Q \quad (1)$$

where h_1 and h_2 are constants and $h_1 >> h_2$ such that $C(Q) > 0$, for all Q.

2. The demand rate R is a function of unit selling price P, and given by $R(P) = kP^{-n}$ (2)
where $k > 0$ is a constant and $n > 0$ is the elasticity of demand. Generally, Selling price P is set as a markup of unit cost C(Q) as

$$P = \alpha\, C(Q) \quad (3)$$

where $\alpha > 1$ is the markup parameter which is a decision variable.

3. Shortages are not allowed. Lead-time is zero. Time horizon is infinite.

4. At each replenishment, a quantity Q is requisitioned. However, the amount received Y is a random variable with

$$E(Y) = bQ \text{ and } V(Y) = \sigma_0^2 + \sigma_1^2 Q^2 \quad (4)$$

where $b > 0$ is the bias factor and σ_0^2 and σ_1^2 are non-negative constants.

5. The inventory holding charge fraction; I per year, the opportunity cost; I_e and replenishment cost A per order are known and constant during the period under consideration.
6. A constant fraction θ ($0 \le \theta < 1$) of on hand inventory gets deteriorated per time unit. There is no repair or replacement of the deteriorated inventory during the period under consideration.

3. NOTATIONS :

- GER(α, Y/Q) = Gross expected revenue when Y – units are received.
- TC(α, Y/Q) = Total cost when Y – units are received.
- NP(α, Y/Q) = Net profit when Y – units are received.
- RI(α, Y/Q) = Residual income when Y – units are received.
- ANP(α, Q) = Average expected net profit.
- ARI(α, Q) = Average expected residual income.

4. MATHEMATICAL MODEL :

A new cycle begins whenever a quantity Q is scheduled for the replenishment. If Y is the quantity received, T(Y/Q) is duration of the cycle and TC(Y/Q) is the total cost for a cycle, then following Shah and Shah (1992)

$$T(Y/Q) = \frac{\theta Y^2}{2R} \text{ and } E(T(Y/Q)) = \frac{\theta\left\{\sigma_0^2 + (\sigma_1^2 + b^2)Q^2\right\}}{2R} \quad (5)$$

TC(α, Y/Q) = $\dfrac{C(Q)(I+\theta)Y^2}{2R} - \dfrac{C(Q)I\theta Y^3}{3R^2} + A$ (6)

GER(α, Y/Q) = $\left\{Y - \dfrac{\theta Y^2}{2R}\right\}(\alpha-1)C(Q)$ (7)

Policy I : AVERAGE EXPECTED NET PROFIT

Total profit generated by the sales of Y – units is (using eqs. (5) and (6))

NP(α, Y/Q) = GER(α, Y/Q) - TEC(α, Y/Q)

$$= \left\{Y - \frac{\theta Y^2}{2R}\right\}(\alpha-1)C(Q) - \left\{\frac{C(Q)(I+\theta)Y^2}{2R} - \frac{C(Q)I\theta Y^3}{3R^2} + A\right\}$$

and hence, expected net profit (using (2) – (4)) is

$$\begin{aligned} ANP(\alpha,Q) = k\alpha^{-n}(\alpha-1)C(Q)^{-n+1}\frac{C(Q)(I+\theta)\sigma_0^2}{2bQ} - \frac{C(Q)(I+\theta)xQ}{2b} + \frac{\theta I\sigma_0^2\alpha^n C(Q)^{n+1}}{k} \\ + \frac{\theta I(\sigma_1^2 + b^2/3)Q^2\alpha^n C(Q)^{n+1}}{k} - \frac{Ak\alpha^{-n}C(Q)^{-n}}{bQ} \\ - \frac{I\theta\sigma_0^4\alpha^n C(Q)^{n+1}}{4kb^2Q^2} - \frac{I\theta x\sigma_0^2\alpha^n C(Q)^{n+1}}{2kb^2} - \frac{A\theta\sigma_0^2}{2b^2Q^2} - \frac{A\theta x}{2b^2} - \frac{I\theta x^2\sigma_0^2\alpha^n C(Q)^{n+1}}{4kb^2} \end{aligned} \quad (8)$$

where $x = \sigma_1^2 + b^2$.

The optimum value of markup $\alpha = \alpha_{01}$ and procurement quantity $Q = Q_{01}$ are obtained by solving

$$\frac{\partial ANP(\alpha,Q)}{\partial\alpha} = E\alpha + F\alpha^{2n} + G = 0 \quad (9)$$

where $E = 4k^2b^2C(Q)Q^2(1-n)$, $F = nI\theta C(Q)^{2n+1}\{4b^2(\sigma_1^2 + b^2/3)Q^4 - \sigma_0^2 - 2x\sigma_0^2Q^2 - x^2Q^4\}$
and $G = 4nbQ\{k2bQC(Q) + Ak2 + \theta I\sigma_0^2\, bQC(Q)^{2n+1}\}$
and

$$\frac{\partial ANP(\alpha,Q)}{\partial Q} = 0 \quad (10)$$

simultaneously using Gauss – Seidal iterative method (Patel (1994)).

The average expected net profit at $\alpha = \alpha_{01}$ and $Q = Q_{01}$ is maximum if and only if

$LM - N^2 < 0,\ L < 0$ (11)

where

L = $\dfrac{\partial^2 ANP(\alpha,Q)}{\partial\alpha^2}, M = \dfrac{\partial^2 ANP(\alpha,Q)}{\partial Q^2}, N = \dfrac{\partial^2 ANP(\alpha,Q)}{\partial\alpha\partial Q}$

POLICY II : AVERAGE EXPECTED RESIDUAL INCOME

Residual income when Y – units are received is given by

$RI(\alpha, Y/Q) = NP(\alpha, Y/Q) - \frac{1}{2} YC(Q)I_e.$

Using eqs. (8) and (4), average expected return on investment per time unit is

$$ARI(\alpha,Q) = k\alpha^{-n}(\alpha-1)C(Q)^{-n+1} - \frac{C(Q)(I+\theta)\sigma_0^2}{2bQ} - \frac{C(Q)(I+\theta)xQ}{2b} + \frac{\theta I\sigma_0^2 C(Q)^{n+1}}{k} + \frac{\theta I(\sigma_0^2 + b^2/3)Q^2\alpha^n C(Q)^{n+1}}{k} - \frac{Ak\alpha^{-n}C(Q)^{-n}}{bQ} - \frac{\theta I\sigma_0^4\alpha^n C(Q)^{n+1}}{4kb^2Q^2} - \frac{\theta Ix\sigma_0^2\alpha^n C(Q)^{n+1}}{2kb^2} - \frac{A\theta\sigma_0^2}{2b^2Q^2} - \frac{AxQ}{2b^2} - \frac{\theta Ix^2Q^2\alpha^n C(Q)^{n+1}}{4kb^2} - \frac{1}{2}bQC(Q)I_e \tag{12}$$

The optimum value of markup $\alpha = \alpha_{02}$ and procurement quantity $Q = Q_{02}$ are by solving eq. (9) and

$$\frac{\partial ARI(\alpha,Q)}{\partial Q} = 0 \tag{13}$$

simultaneously using Gauss – Seidal iterative method (Patel (1994)). The average expected residual income at $\alpha = \alpha_{02}$ and $Q = Q_{02}$ is maximum if and only if eq. (11) holds for $ARI(\alpha, Q)$.

The interdependence of various parameters on average expected net profit and average expected residual income has been studied in the following numerical illustration.

5. NUMERICAL ILLUSTRATION :

Consider following data in proper units :

$[k, h_1, h_2, I, b, A, \sigma_0^2, \sigma_1^2] = [3375000, 100, 0.01, 0.24, 0.75, 250, 5.0, 0.1]$

Table 1 Effect of deterioration and elasticity

θ n		0.01	0.02	0.03
1.8	α Q ANP	2.299 153.74 23758.59	2.303 142.40 23693.79	2.307 133.20 23633.20
2.0	α Q ANP	2.082 81.57 7776.06	2.087 77.29 7740.50	2.091 73.62 7706.59
2.2	α Q ANP	1.961 48.02 2503.63	1.967 45.78 2482.15	1.974 43.76 2461.54

Table 2 Effect of deterioration and elasticity

θ n		0.01	0.02	0.03
1.8	α Q ANP	2.300 151.02 23744.53	2.304 140.22 23680.72	2.307 131.46 23620.97
2.0	α Q ANP	2.083 81.57 7768.53	2.088 76.43 7733.35	2.092 72.87 7699.77
2.2	α Q ANP	1.962 47.50 2499.18	1.969 45.29 2477.90	1.975 43.35 2457.47

6. CONCLUSIONS :

From Table 1, we observe that as elasticity, n increases, the markup α, the optimum order quantity Q and average expected net profit decreases. i.e. this model should be used for products with smaller elasticity of demand. Furthermore, as deterioration rate, θ of units in inventory increases, the optimum markup α increases but average expected net profit and optimum purchase quantity decreases. Thus, this model will be useful for items with any rate of deteriorating having smaller elasticity of demand for controlling the inventory.

Similar conclusions can be made when residual income is used as a measure of the effectiveness. However, comparing these two policies, it can be seen that

1. The optimum value of markup α is slightly higher in policy 2 as compared to policy 1.
2. The optimum purchase quantity Q is slightly lower in policy 2 as compared to policy 1.

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