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Numerical simulation of flood events in Santo Antônio de Pádua through the Saint-Venant equations
Simulación numérica de eventos de inundación en Santo Antônio de Pádua usando ecuaciones de Saint-Venant

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Resumen Este artículo presenta un análisis de inundaciones en la ciudad de Santo Antônio de Pádua, ubicada en la región noroeste de Río de Janeiro. El objetivo es modelar las inundaciones mediante las ecuaciones de Saint-Venant, con base en datos recopilados y puestos a disposición por el Instituto Estatal del Medio Ambiente, para las condiciones iniciales y de borde, y por la Agencia Nacional de Aguas y Saneamiento, para verificar la eficiencia del experimento. Para ello, las ecuaciones se resolverán mediante el Método de Diferencias Finitas, de modo que se puedan comparar los datos experimentales en un tramo del río Pomba, cuyos resultados se aproximaron significativamente a los datos reales.

Palabras Clave: ecuaciones de Saint Venant, método de diferencias finitas, modelización de inundaciones, Santo Antônio de Pádua.

Abstract *This paper presents an analysis of flooding events in the city of Santo Antônio de Pádua, located in the region of Noroeste Fluminense. The objective is to perform the modeling of flooding events from the Saint-Venant equations based on data collected and made available by the State Environmental Institute, for the initial and boundary conditions, and by the National Water and Basic Sanitation Agency, to verify the efficiency of the experiment. For this, the equations will be solved using the Method of Finite Differences, so that comparison can be made between the experimental data in a stretch of the Pomba River, whose results were significantly close to the real data.*

Keywords: *Saint Venant equations, finite difference method, flood modeling, Santo Antônio de Pádua.*

Mathematics Subject Classification: 90B06, 90C59, 68T20, 90C27.

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Introduction

Hydrological planning is fundamental for the development of countries [11], especially for nations that constantly suffer from floods, making it necessary to mitigate future problems. According to [10], floods generally occur due to physical environmental constraints, which can be intensified by basin occupation.

Flood-related catastrophes have increased since the 2000s compared to the two previous decades, according to the World Meteorological Organization (WMO) [9]. These natural disasters occur when waters overflow their drainage bed, whether natural or artificial [13]. Although conditioned by natural factors, flood events have intensified globally due to population growth and urbanization, which modifies land use conditions and causes serious consequences.

Data released by the Intergovernmental Panel on Climate Change (IPCC) [8] reinforce the fact that rising global temperatures alter rainfall levels across the globe, since the greenhouse effect causes more moisture to be drawn into the water cycle, resulting in more precipitation events which, on some occasions, remain concentrated in certain locations. According to a report from the first half of 2022 issued by the Centre for Research on the Epidemiology of Disasters (CRED) [5], during the months of February and May of 2022, Brazil recorded approximately 272 and 116 deaths resulting from floods, respectively, appearing twice among the 10 largest disasters in terms of number of deaths, which clearly demonstrates the suffering of the Brazilian population with these natural disasters, as shown in Table 1.

Table 1. Top-10 disasters by number of deaths [*Las 10 mayores catástrofes por número de muertes*].

Disaster	Month	Country	Total deaths
Flood	May-August	India	1354
Flood	June-September	Pakistan	1061
Earthquake	June	Afghanistan	1036
Flood	April	South Africa	501
Storm/Megi	April	Philippines	289
Flood	February	Brazil	272
Storm/Batsirai	February	Madagascar	121
Flood	May	Brazil	116
Flood	May-June	Bangladesh	82
Flood	June-August	Niger	75

The municipality of Santo Antônio de Pádua - RJ, for example, has been constantly suffering from floods [14]. A recent example occurred at the beginning of January 2022, when the river level rose to 1.65 meters above its overflow threshold, causing major disruptions for residents living in flood-prone areas along the banks of the Pomba River, who are generally considered to have low income. Among the affected areas are the municipal hospital, the city hall's administrative offices, schools, a paper and pulp industry, and the town center, where commercial enterprises are concentrated.

Understanding these flood events becomes essential when considering the mitigation of problems caused to the population. A literature review identified a scarce number of studies on such events in Santo Antônio de Pádua (Table 2).

In order to understand and characterize the flood events in Santo Antônio de Pádua, it is proposed to perform simulations on the Pomba River using the Saint-Venant equations that describe the flow of rivers and channels. To solve the problem, the Finite Difference Method will be employed, which uses the approximation of derivatives by finite differences.

Table 2. Literature review summary [*Resumen de la revisión de la literatura*].

Topic	Article	Monograph	Total
Fissured Aquifers	1	0	1
Productive Arrangement	1	0	1
Direct Revenue Collection	1	0	1
Climatic Characterization	1	1	2
Thermobarometric Data	1	0	1
Mineral Water Sources	2	0	2
Polluting Sources	0	1	1
History of the Raul Veiga Bridge	1	0	1
Flood	1	2	3
Child/Adolescent Malocclusion	1	0	1
Mining	1	0	1
Retaining Wall	1	0	1
Digital Museum	0	1	1
African Snail Population	1	0	1
Water Quality	1	1	2
Ornamental Rock Waste	8	0	8
Pomba River and Education	3	1	4
Total	25	7	32

Study highlights

The relevance of this study lies in the need to understand and mitigate the impacts of floods in vulnerable urban areas, such as the municipality of Santo Antônio de Pádua - RJ, which is frequently affected by extreme hydrological events. The scarcity of specific studies in the region highlights a scientific gap, considering the recurring social, economic, and environmental damages. Through mathematical modeling of surface runoff using the Saint-Venant equations and the Finite Difference Method, it is possible to simulate flood scenarios in the Pomba River and analyze the hydrodynamic behavior of the local fluvial system. This quantitative approach provides technical support for hydrological planning and the development of flood control strategies. In addition, the study contributes with the potential for replication in other river basins with similar characteristics. Understanding the variables involved in the flooding process enables the optimization of containment works, the direction of public policies, and the minimization of human and material losses, contributing in a scientific, technical, and social manner.

1. Study area

Santo Antônio de Pádua is a municipality in the state of Rio de Janeiro. It is located at 21°32′22" south latitude and 42°10′49" west longitude, with an altitude of 86 meters. Its population, as recorded in 2010, was 40,589 inhabitants, and the estimated population in 2020 was 42,705 inhabitants. The municipality has an area of 603.633 km^2, subdivided into the districts of Santo Antônio de Pádua (the seat), Baltazar, Santa Cruz, Campelo, Marangatu, Monte Alegre, Paraoquena, São Pedro de Alcântara, and Ibitiguaçu.

The city is supplied by the Pomba River, which flows through it, and is one of the main tributaries on the left bank of the Paraíba do Sul River. Santo Antônio de Pádua has already suffered from several floods that submerged many neighborhoods, inundating homes, factories, schools, hospitals, and leaving thousands of people displaced.

In 1979, the city was affected by the flooding of the Pomba River, with no records of the level reached or the number of displaced or homeless people. Decades later, the largest recorded flood occurred in 2008, when the Pomba River rose more than 4.5 meters above its normal bed, destroying one of the bridges giving access to the Bairro Cidade Nova and part of the pedestrian bridge, also called the "iron bridge". According to Civil Defense data, more than 54,000 people were displaced, and over 11,700 were homeless. The damages caused by the flood amounted to R$ 30 million.

Since 2010, a dam has been in operation in the city of Palmas-MG, located approximately 36km upstream from the urbanized area of Santo Antônio de Pádua. This dam controls the flow of the Pomba River, and when the reservoirs reach high levels due to excessive rainfall in the region, the water volume is gradually released, causing flooding in the urbanized area of Pádua. Furthermore, this volume increases due to increased flow in the tributaries that drain the water into the microbasins in the region. Added to this fact is the occupation of floodplains by riverside populations, which are constantly affected by rising water levels in the region.

In 2012, a new flood affected the municipality. According to G1, more than 12,000 people were displaced, causing damages of R$ 20 million. In early 2014, the river level reached 4.7 meters, but did not flood the streets since the overflow threshold is 5 meters, according to INEA. In June of the same year, flood prevention works were announced, including the removal of stones (rock removal) and dredging (sand and mud removal), drainage, and urbanization along a long stretch of the Pomba River.

In 2020, Santo Antônio de Pádua was once again affected by floods, with three floods occurring in January, February, and March, with levels surpassing 6 meters above the riverbed in the first two - January and February. In 2022, the city experienced flooding in January (from January 8th to 13th), when the Pomba River reached a maximum level of 6.45 meters, exceeding the overflow threshold (5 meters), flooding streets and homes in several neighborhoods, affecting about 12,000 people.

According to the city hall, approximately 3,000 people were displaced, and 24 were left homeless. Consequently, in February, the Pomba River again exceeded the overflow threshold, reaching 5.20 meters according to the INEA website. On this occasion, the Campelo district was the most affected due to the overflow of the Santo Antônio Stream in the city of Miracema. Also, on February 16th, the Pomba River reached 5.61 meters, flooding several neighborhoods once again.

2. Mathematical modeling of the proposed problem

The study of open channel flows is an important topic in the fields of Hydraulics and Hydrology, as it is extremely relevant to understand the behavior of a river in order to make predictions and future scenarios.

Mathematically modeling a real phenomenon typically requires dealing with differential equations, which involve derivatives of the unknowns. In the context of open channel hydraulics, the commonly used equations are the Saint-Venant equations, which form the basis for the development of models employed to represent unsteady and non-uniform surface runoff in basins, rivers, and unidirectional free-surface channels [2, 15].

The Saint Venant equation is derived from two equations. The first equation is the Continuity equation. In the application to rivers, as the original principle refers to mass conservation, the volumes of water are multiplied by the specific mass, so that the resulting balance is made with mass terms through the same control volume [1]. The second equation, the Momentum equation, is based on a summation of forces acting on a control volume, which is equal to the sum of the rate of change of momentum within the control volume and the rate of momentum flux through the control surface.

These equations (Continuity and Momentum, respectively) are formally described as [12]:

$$\frac{\partial Q}{\partial x}+\frac{\partial A}{\partial t}+q=0, \tag{1}$$

$$\frac{\partial Q}{\partial t}+\frac{\partial}{\partial x}\left(\beta\frac{Q^2}{A}\right)+Ag\left(\frac{\partial h}{\partial x}-S_0+S_f+S_e\right)-\beta q V_x+WB=0, \tag{2}$$

where $x\,[\mathrm{m}]$ and $t\,[\mathrm{s}]$ are the spatial and temporal variables, respectively; $Q\,[\mathrm{m}^3/\mathrm{s}]$ is the channel discharge; $A\,[\mathrm{m}^2]$ is the cross-sectional area; $q\,[\mathrm{m}^2/\mathrm{s}]$ is the lateral inflow per unit length; $\beta\,[-]$ is the Boussinesq coefficient; $g\,[\mathrm{m}/\mathrm{s}^2]$ is the gravitational acceleration; $h\,[\mathrm{m}]$ is the water depth; $S_0\,[\mathrm{m}/\mathrm{m}]$ is the bed slope of the channel; $S_f\,[\mathrm{m}/\mathrm{m}]$ is the friction slope; $S_e\,[-]$ is the head loss due to vortices; $V_x\,[\mathrm{m}/\mathrm{s}]$ is the velocity of lateral inflow; $W\,[\mathrm{m}^3/\mathrm{s}^2]$ is the wind resistance coefficient; and $B\,[\mathrm{m}]$ is the free surface width.

In equation (2), the bed slope of the channel, the energy slope, and the head loss due to vortices are expressed as:

$$S_0=-\frac{\partial z_f}{\partial x}, \tag{3}$$

$$S_f=\frac{Q^2n^2}{R^{4/3}A^2}, \tag{4}$$

$$S_e=\frac{K_e}{2g}\frac{\partial}{\partial x}\left(\frac{Q}{A}\right)^2, \tag{5}$$

where $z_f[m]$ is the elevation of the riverbed; $n[m^{1/3}s]$ is Manning's roughness coefficient; $R[m]$ is the hydraulic radius; and K_e is a dimensionless coefficient, negative for expansion and positive for contraction.

The Saint Venant equations are hyperbolic partial differential equations with analytical solutions only in very specific cases. Therefore, in this work, the Finite Difference Method is proposed, as it is a relatively simple numerical solution method with acceptable error margins [4]. Furthermore, these equations can be simplified by eliminating some terms from the Momentum equation, in order to obtain a simpler model formulation and provide an advantage in computational efficiency. For this purpose, the inertial model formulation presented by [3] is adopted, which neglects only the advective inertial term, and can be formally described as:

$$\frac{\partial Q}{\partial t} + Ag\left(\frac{\partial h}{\partial x} - S_0 + S_f\right) = 0, \tag{6}$$

where $h[m]$ is the flow depth.

By substituting the pressure term, given by $Ag\left(\frac{\partial h}{\partial x}\right)$, and the weight term, equation (3), equation (6) can be rewritten as:

$$\frac{\partial q}{\partial t} + gh\frac{\partial y}{\partial x} + g\frac{|q|qn^2}{h^{7/3}} = 0. \tag{7}$$

3. Discretization of the proposed problem

The Finite Difference Method (FDM) aims to solve differential equations by approximating derivatives using finite differences [7]. The method employs problem discretization to approximate the function at each discretization point, meaning that the solutions obtained through the finite difference method will always be discrete. If it is necessary to determine the function's value between discretization points, interpolation methods are used to estimate the corresponding function value [6]. The resolution of the problem using the finite difference method involves the following steps: (*i*) discretization of the proposed problem; (*ii*) construction of the discrete problem (which transforms the proposed problem into a system of equations); and (*iii*) resolution of the discrete problem.

By adopting the Finite Difference Method, equation (1) is discretized using a spatially centered and time-forward formulation, thus the following finite difference approximations are applied to the derivatives:

$$\frac{\partial h}{\partial t} \cong \frac{h_i^{k+1} - h_i^k}{\Delta t}, \tag{8}$$

$$\frac{\partial q}{\partial x} \cong \frac{q_{i+\frac{1}{2}}^{k+1} - q_{i-\frac{1}{2}}^{k+1}}{\Delta x}, \tag{9}$$

where the approximation errors are of order $O(\Delta t)$ and $O(\Delta x)$, respectively. The indices k and i refer, respectively, to the time and space positions, with $t = k\Delta t$ and $x = i\Delta x$.

In this configuration, the mesh points $i - 1/2$ and $i + 1/2$ are intermediate points and will be addressed later. Thus, based on the discretizations presented in equations (8) and (9), equation (1) is approximated by:

$$\frac{h_i^{k+1} - h_i^k}{\Delta t} + \frac{q_{i+\frac{1}{2}}^{k+1} - q_{i-\frac{1}{2}}^{k+1}}{\Delta x} = 0. \tag{10}$$

The equation (7) is discretized with the space-centered and time-advancing formulation, given by equations (11) and (12):

$$\frac{\partial y}{\partial x} \cong \frac{y_{i+1}^k - y_i^k}{\Delta x}, \tag{11}$$

$$\frac{\partial q}{\partial t} \cong \frac{q_{i+\frac{1}{2}}^{k+1} - q_{i+\frac{1}{2}}^k}{\Delta t}, \tag{12}$$

where the approximation errors are of the order of $O(\Delta x)$ and $O(\Delta t)$, respectively.

Therefore, the substitutions of equations (11) and (12) into equation (7) lead to equations (13):

$$\frac{q_{i+\frac{1}{2}}^{k+1} - q_{i+\frac{1}{2}}^k}{\Delta t} + g \cdot h_{i+\frac{1}{2}}^k \cdot \frac{y_{i+1}^k - y_i^k}{\Delta x} + g \cdot \frac{|q_{i+\frac{1}{2}}^k| \cdot q_{i+\frac{1}{2}}^{k+1} \cdot n^2}{\left(h_{i+\frac{1}{2}}^k\right)^{\frac{7}{3}}}, \tag{13}$$

where the term $h_{i+\frac{1}{2}}^k$ refers to the depth of the cross-section between segment i and segment $i+1$, which is estimated by equation (14), an adaptation of the proposal by [3]:

$$h_{i+\frac{1}{2}}^k = \max\{y_i^k, y_{i+1}^k\} - \max\{z_i, z_{i+1}\}, \tag{14}$$

where $h_{i+\frac{1}{2}}^k$ represents the depth at the section that divides the subsegments i and $i+1$, y_i^k is the water level elevation in subsegment i, y_{i+1}^k is the water level elevation in subsegment $i+1$, z_i is the riverbed elevation in subsegment i, and z_{i+1} is the riverbed elevation in subsegment $i+1$.

Note that, in equation (13), at each time interval k, the values at the beginning of the interval are known, while the values of the variables with index $k+1$ are unknown. Therefore, by making explicit the term with index $k+1$, we obtain equation (15):

$$q_{i+\frac{1}{2}}^{k+1} = \frac{\left(\left(q_{i+\frac{1}{2}}^k\right) - g \cdot \Delta t \cdot \left(h_{i+\frac{1}{2}}^k\right) \cdot \frac{(y_{i+1}^k - y_i^k)}{\Delta x}\right)}{\left(1 + \frac{g \cdot \Delta t \cdot \left(\left|q_{i+\frac{1}{2}}^k\right|\right) \cdot n^2}{\left(h_{i+\frac{1}{2}}{}^k\right)^{\frac{7}{3}}}\right)}. \tag{15}$$

Given an initial condition where the water levels in all subsections i are known, the solution for the water levels and flow rates at the end of the time interval is initially found using equation (14), then equation (15) is used to find the flow per unit width at each section. Based on the flow values $q_{i+\frac{1}{2}}^{k+1}$ obtained, equation (16) is applied, which is a reorganization of equation (10) aimed at making the term h_i^{k+1} explicit, thus allowing the calculation of the depth for the subsections i at the end of the time interval:

$$h_i^{k+1} = h_i^k - \frac{\Delta t}{\Delta x} \cdot \left(q_{i+\frac{1}{2}}^{k+1} - q_{i-\frac{1}{2}}^{k+1} \right). \qquad (16)$$

Finally, the values of the water level elevations in all subsections are obtained using equation (17):

$$y_i^{k+1} = z_i + h_i^{k+1}. \qquad (17)$$

Thus, the solution to the problem is found by repeatedly applying equations (14), (15), (16), and (17). For solving the flood propagation problem in a river, one upstream boundary condition and one downstream boundary condition are required. Typically, the upstream boundary condition used is the imposition of a flow hydrograph at the first section, where, in this approach, it is assumed that the value of q_{i+1}^k at the first subsection is known for all time intervals k. Regarding the downstream boundary condition, a time series of water level elevations at the last section of the river is used, which means that the values of h_i^k and y_i^k at the last subsection are known for all time intervals k.

Furthermore, as can be seen from the adopted numerical scheme, we have the representation of an explicit model, which is subject to a strict time step size restriction to avoid numerical instability. Thus, the choice of time step Δt must comply with the Courant-Friedrichs-Levy condition, presented in equation (18):

$$(\sqrt{g \cdot h}) \cdot \frac{\Delta t}{\Delta x} \leq 1, \qquad (18)$$

where g is the acceleration due to gravity (m/s^2), h is the depth (m), Δx is the length of the river subsection (m), and Δt is the time step for calculation (s).

Thus, the calculation time step must satisfy equation (18):

$$\Delta t = \alpha \cdot \frac{\Delta x}{\sqrt{g \cdot h}}, \qquad (19)$$

where α is a value less than or equal to 1. However, it is advisable that the value of α be between 0.3 and 0.7 in order to avoid numerical instability in the solution of equations (14), (15), (16), and (17) [3].

4. Results and discussion

This work focuses on the case study of the Pomba River in the municipality of Santo Antônio de Pádua - RJ. Thus, the INEA database was considered as the source of information for the initial conditions and boundary conditions of the experiment. The ANA database was also used to verify the efficiency of the model applied in this work, performing a comparison between the experimental data in a certain stretch of the river and the data from an ANA station.

Although it is crucial to adjust the mathematical model so that it satisfactorily represents and characterizes the entire modeled flood event, this initial investigation aimed to model only the peak of the flood wave, focusing primarily on studying the influence of velocity on the event of interest. Factors such as a better characterization of the riverbed, which features rocky regions, as well as the simplification of the model due to the one-dimensional approach of the Saint-Venant equations, are some of the reasons that led to a greater discrepancy between the numerical results before and after the peak of the flood wave compared to the experimental data used as a reference in the calibration process.

In Figure 1, a simplified version of the river stretch used for the experiments is presented, where the initial point (coordinates 21°30’40"S and 42°12’25"W), final point (coordinates 21°32’57"S and 42°10’37"W), monitoring point (which corresponds to the ANA station that will be used to verify the model), and the river trajectory from the initial point to the final point are indicated. The total length of this trajectory is 5600 meters.

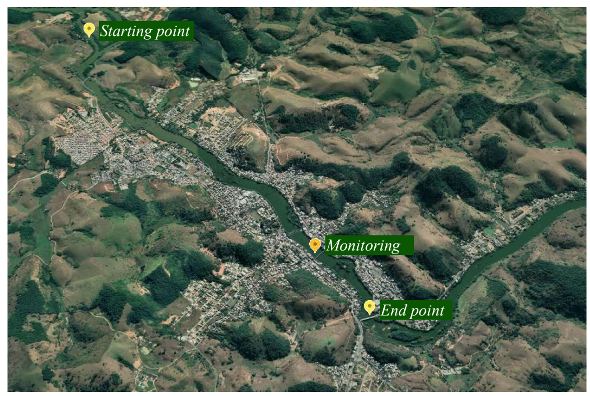


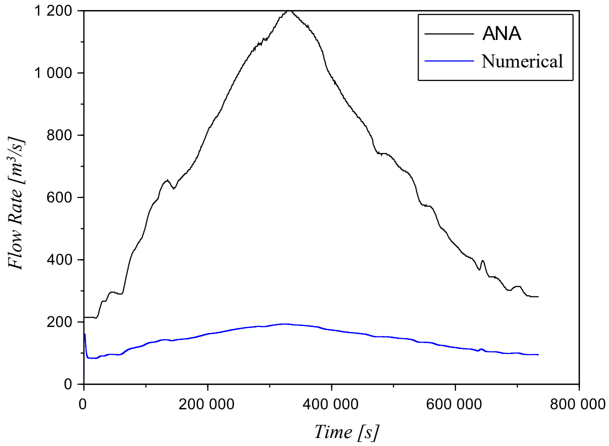
Figure 1. Representation of the stretch adopted for the experiment using Google Earth Pro [*Representación del tramo adoptado para el experimento utilizando* Google Earth Pro].

The initial objective is to determine a more consistent and realistic value for the river’s velocity, while for the other parameters of interest, standard values were considered. The experiments used river velocities ranging from 0.2 m/s^2 to 1.8 m/s^2. Additionally, 114 nodes were used in the spatial mesh to divide the river stretch for the application of the Finite Difference Method.

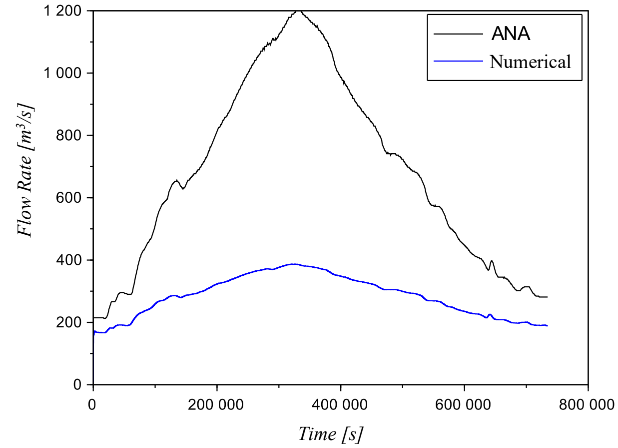
Figures 2 and 3 show the comparison between the flow over time obtained in the simulation and the experimental data extracted from the ANA database, for the monitoring point of interest.

Based on the results for flow over time (Figure 2), it is initially observed that the velocity ranging from 0.2 m/s to 0.6 m/s does not yield good results, as the flow curves over time show that the model could not "adapt" to the real data. However, for a velocity of 0.6 m/s, the model adequately represents the beginning and end of the flood wave, although its peak is still misrepresented.

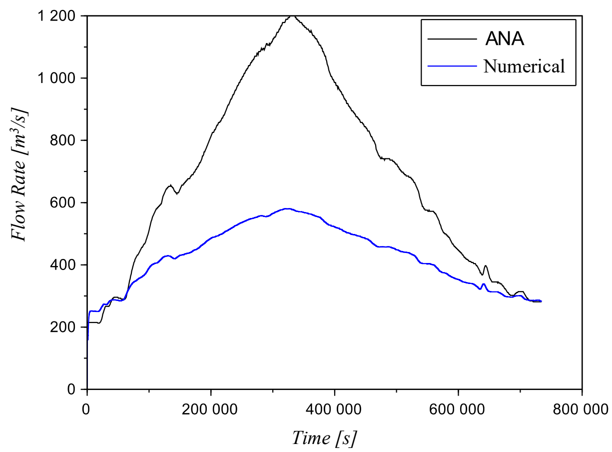
On the other hand, simulations considering velocities ranging from 0.6 m/s to 1.2 m/s tend to increase the numerical



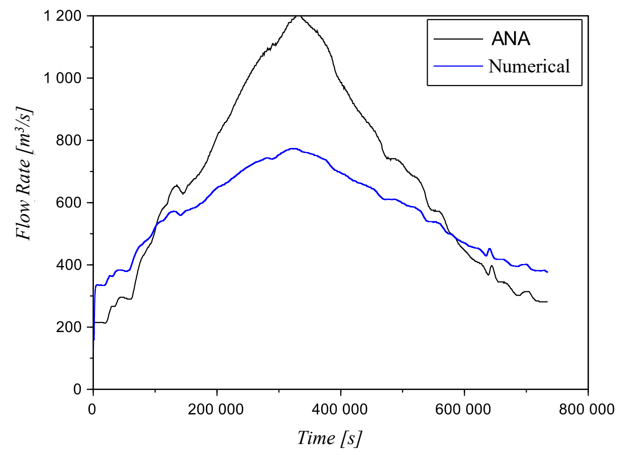
(a) Flow rate over time for velocity equal to 0.2 m/s



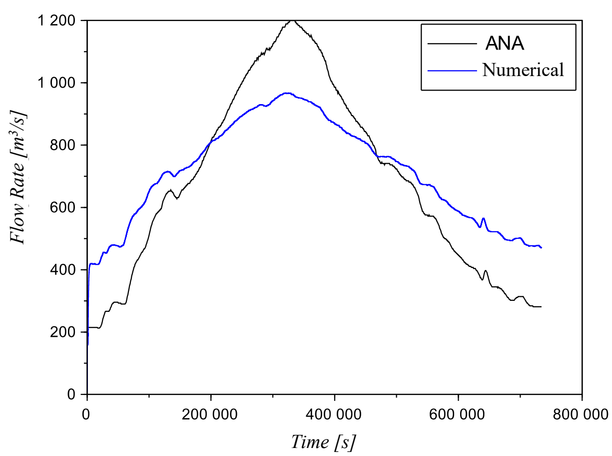
(b) Flow rate over time for velocity equal to 0.4 m/s



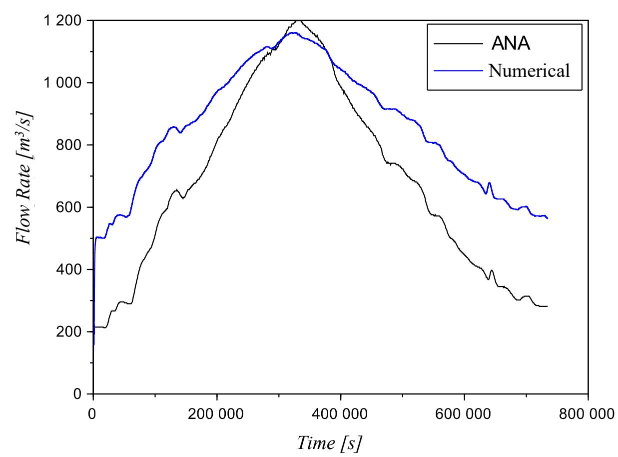
(c) Flow rate over time for velocity equal to 0.6 m/s



(d) Flow rate over time for velocity equal to 0.8 m/s

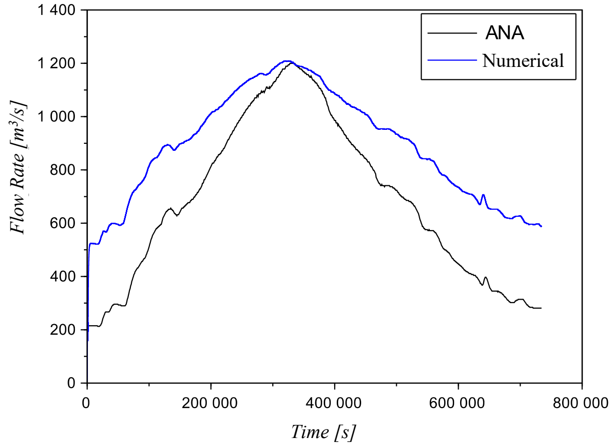


(e) Flow rate over time for velocity equal to 1.0 m/s

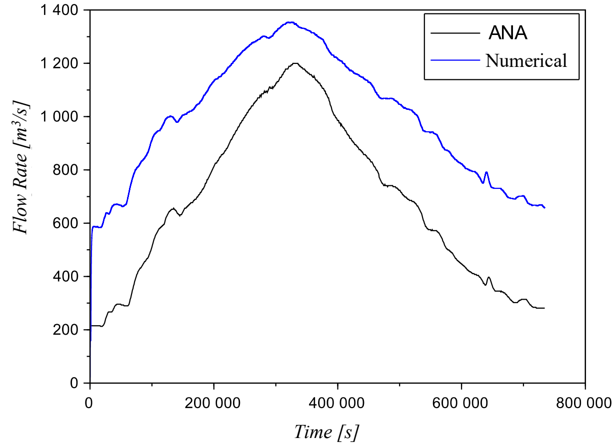


(f) Flow rate over time for velocity equal to 1.2 m/s

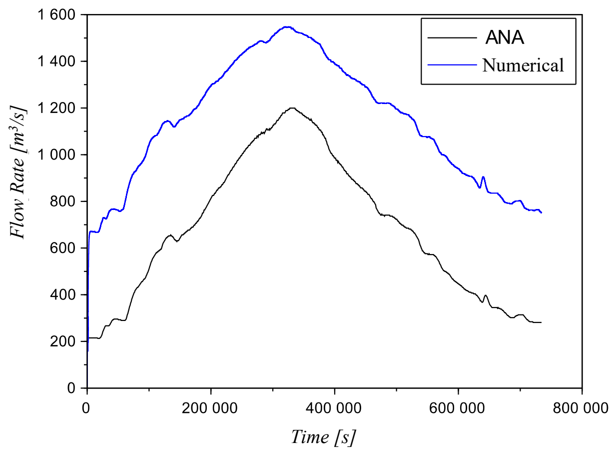
Figure 2. Flow rate over time for the velocity variation up to $1.2\ \mathrm{m/s^2}$ [*Caudal a lo largo del tiempo para la variación de velocidad hasta* $1{,}2\ \mathrm{m/s^2}$].



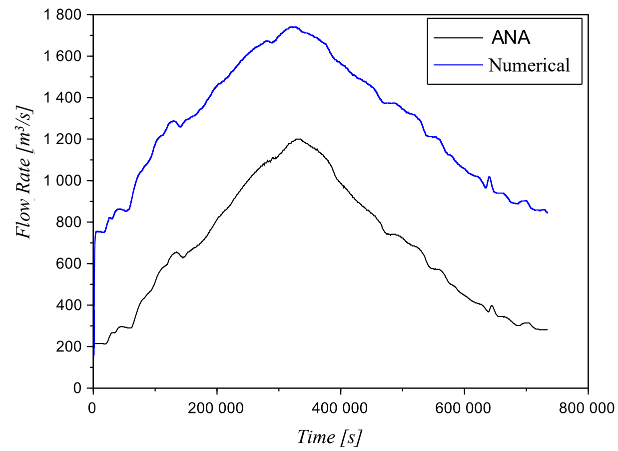
(a) Level over time for velocity equal to $1,25\ m/s$.



(b) Level over time for velocity equal to $1,4\ m/s$.



(c) Level over time for velocity equal to $1,6\ m/s$.



(d) Level over time for velocity equal to $1,8\ m/s$.

Figure 3. Water level over time for the variation in velocity up to $1,8\ m/s$ [*Nivel del agua a lo largo del tiempo para la variación de velocidad de hasta* $1,8\ m/s$].

results of the flood wave, particularly in terms of its peak flow, bringing them closer to the ANA data, demonstrating that the model proposed in this work tends to provide good approximations in this region.

Regarding velocities above $1.2\ m/s$ (Figures 3), it is observed that the numerical solution successfully approximated the real data curve at the peak of the flood wave, but the flow values over time are distant from the real data when the velocity exceeds $1.2\ m/s$. Therefore, based on these experimental results, it can be concluded that the best velocity approximation for modeling the peak flood flow over time corresponds to a velocity of $1.25\ m/s$, as shown in Figure 3(a).

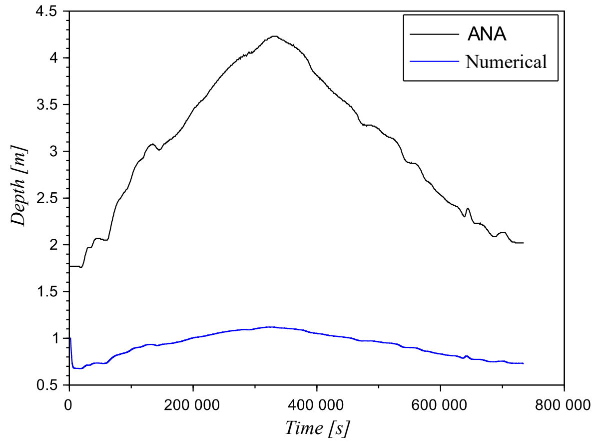
Next, Figures 4 and 5 present the depth over time obtained from the numerical solution in comparison with the depth over time extracted from the ANA database.

Based on the results obtained for the water level at the monitoring station (Figures 4 and 5), it is observed that for velocities between $0.2\ m/s$ and $0.8\ m/s$, the solution of the mathematical model did not yield significant results, as it approximated the real data relatively poorly.

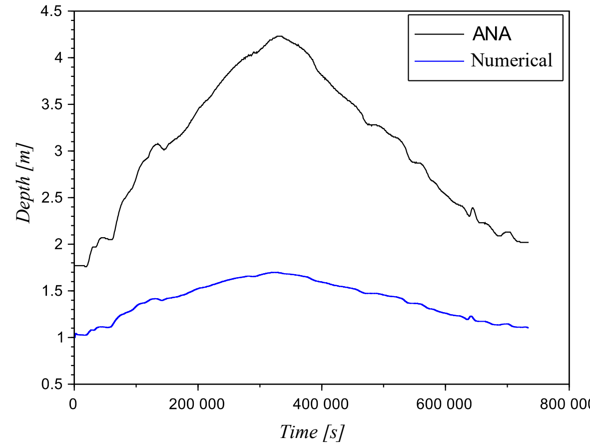
For velocities between $1.0\ m/s$ and $1.25\ m/s$, it is observed that for time intervals approximately below 200000 s and above 450000 s, the model provided good approximations to the curve representing the real data. However, within these time intervals, it failed to properly characterize the peak level of the real data.

On the other hand, with velocities above $1.2\ m/s$, particularly for a velocity of $1.80\ m/s$, it is observed that the numerical model better approximates the "peak" of the real data curve. It is also noted that the numerical solution of the mathematical model maintains a good characterization of the initial and final parts of the experiment. However, from $1.4\ m/s$ onwards, the model produces values for the water level that are higher than the real data.

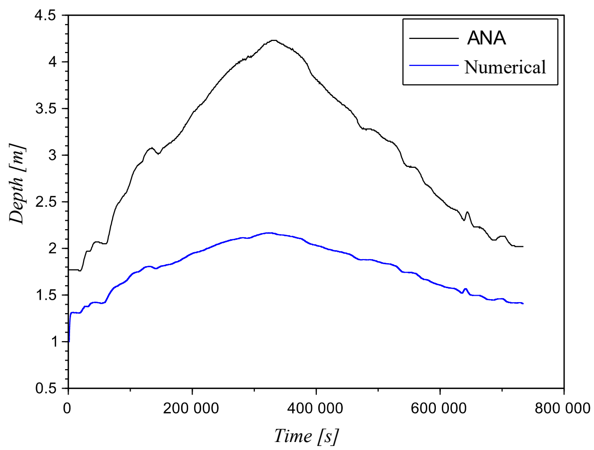
Therefore, from the comparison between numerical results and experimental data, it can be concluded that the best velocity approximation for the water level over time corresponds to a velocity of $1.8\ m/s$, as shown in Figure 5(d).



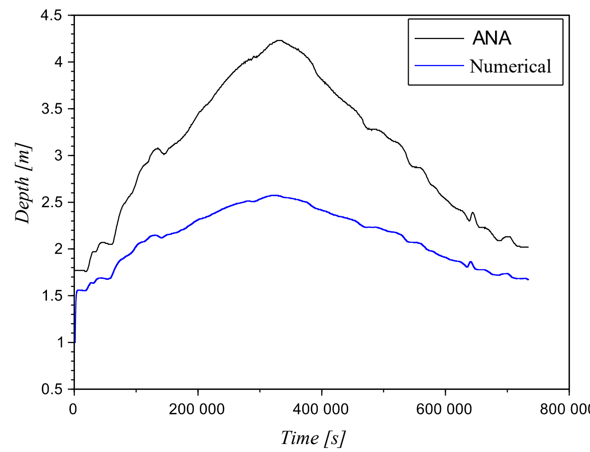
(a) Level over time for velocity equal to $0,2\ m/s^2$.



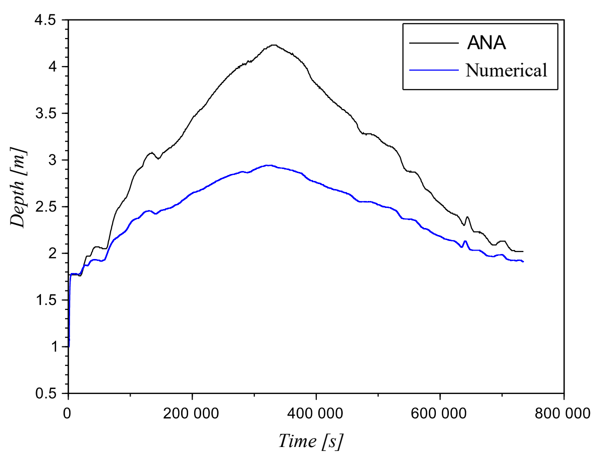
(b) Level over time for velocity equal to $0,4\ m/s^2$.



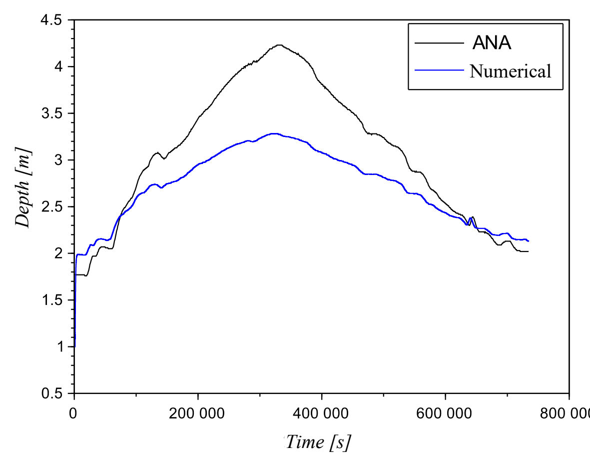
(c) Level over time for velocity equal to $0,6\ m/s^2$.



(d) Level over time for velocity equal to $0,8\ m/s$.

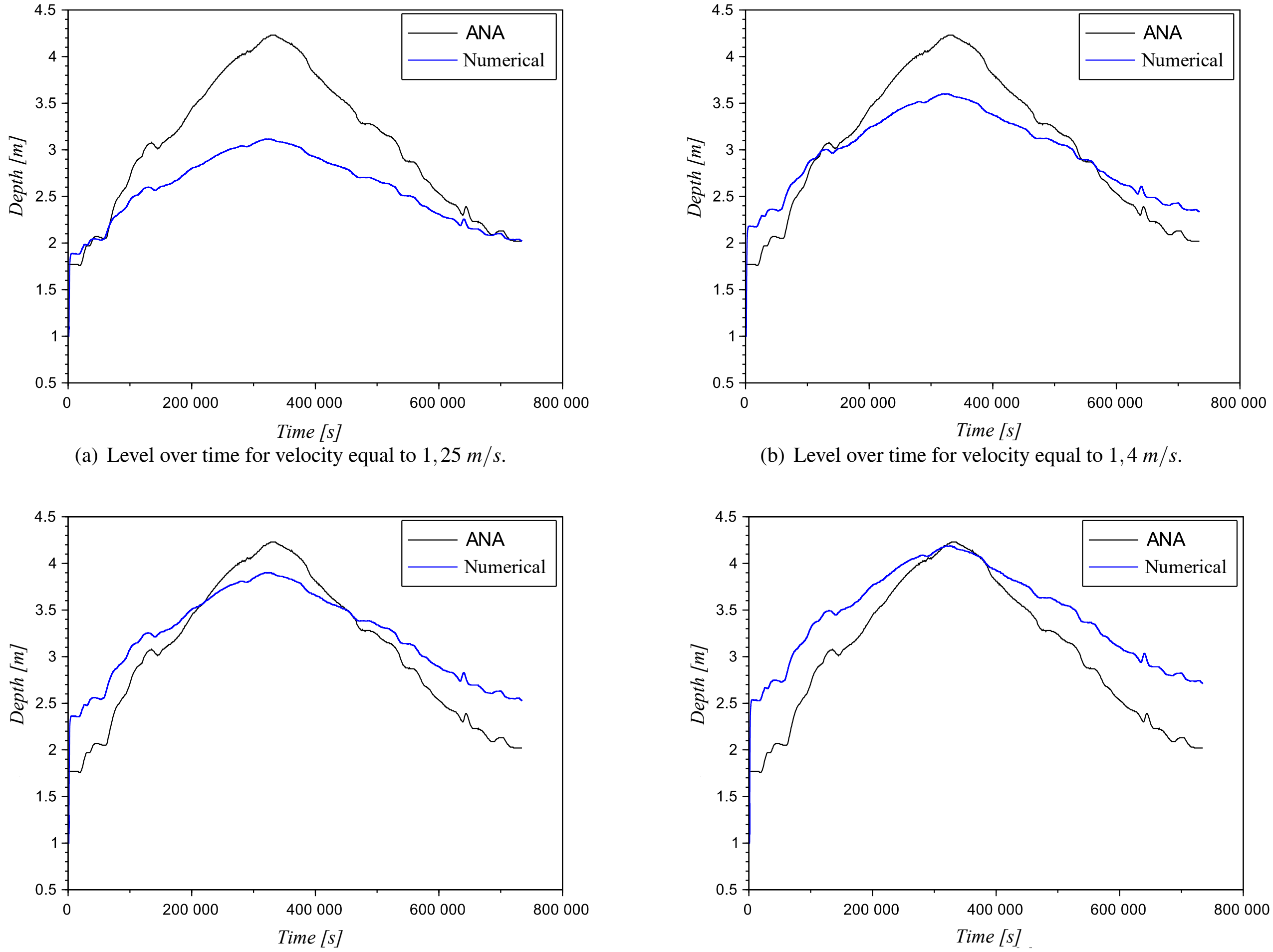


(e) Level over time for velocity equal to $1,0\ m/s$.



(f) Level over time for velocity equal to $1,2\ m/s$.

Figure 4. Water level over time for the variation in velocity up to $1,2\ m/s$ [*Nivel del agua a lo largo del tiempo para la variación de velocidad de hasta* $1,2\ m/s$].



(a) Level over time for velocity equal to $1,25\ m/s$.

(b) Level over time for velocity equal to $1,4\ m/s$.

(c) Level over time for velocity equal to $1,6\ m/s$.

(d) Level over time for velocity equal to $1,8\ m/s$.

Figure 5. Level over time for velocity up to $1,8\ m/s$ [*Nivel a lo largo del tiempo para velocidades de hasta* $1,8\ m/s$].

5. Conclusions, recommendations and future work

Considering the scenario of Santo Antônio de Pádua - RJ and the flooding events that occur constantly, bringing great distress to the local population, this paper used the Finite Difference Method to solve the Saint-Venant equations, which model the flow in rivers.

Based on the results obtained, it can be concluded that the mathematical model found the best velocity approximation for discharge and water level over time as $1.25\ m/s$ and $1.8\ m/s$, respectively. These results were minimally satisfactory, as the simulated data are relatively close to the data collected by ANA. However, further studies can be conducted to improve the obtained results, aiming to get even closer to real data by refining the parameters or even using other numerical methods.

Conflicts of interest

It is declared that there are no conflicts of interest.

Supplements

It is declared that there are no supplements for this paper.

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Visualization G.S.G., J. V. S. L., M. T. F., F.F.F., W.R.T.

Writing - original draft G.S.G., J. V. S. L., M. T. F., W.R.T.

Writing - review & editing F.F.F., W.R.T.

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