SOME FAMILIES OF ESTIMATORS USING TWO AUXILIARY VARIABLES IN STRATIFIED RANDOM SAMPLING

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ABSTRACT
This paper suggests some generalized classes of modified ratio, regression-cum-ratio and exponential ratio type estimators for finite population mean of the study variable utilizing the information on two auxiliary variables in stratified random sampling. The expressions for bias and mean square error of proposed class of estimators have been derived and compare with those of existing estimators. An empirical study is carried out to demonstrate the efficiency of the proposed class of estimators over others and it is found that the empirical results support the theoretical study.

KEYWORDs: Auxiliary information, bias, efficiency, mean square error, regression estimator.

MSC: 62D05

RESUMEN
En este trabajo se proponen algunas clases generalizadas de estimadores identificadas de razón, cum.regresión-cum razón y estimadores del tipo razón exponenciales para la media de una población finita de una variable de estudio que utiliza la información de dos variables – auxiliares en muestreo estratificado aleatorio. Las expresiones del sesgo y el error cuadrático medio de la clase de estimadores propuesta han sido derivadas y comparadas con los de otros estimadores existentes. Un estudio empírico se desarrolló para demostrar la eficiencia de las clases de estimadores propuestas. Estos soportan el estudio teórico

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1. INTRODUCTION

In survey sampling, it is usual to make use of the auxiliary information at the estimation stage in order to improve the precision or accuracy of an estimator of unknown population parameter of interest. Ratio, product and regression methods of estimation are good examples in this context. Diana (1993) suggested a class of estimators of the population mean using one auxiliary variable in the stratified random sampling and examined the mean square Error (MSE) of the estimators up to the k^{th} order of approximation. Kadilar and Cingi (2003), Singh and Vishwakarma (2008), Koyuncu and Kadilar (2009), Singh et. al. (2013) proposed estimators in stratified random sampling. Ghosh (1958) has suggested estimators in stratified random sampling with multiple characteristics. Also some other authors including Upadhyaya and Singh (1999), Kadilar and Cingi (2005), Khoshnevisan et al. (2007), Singh et al. (2008a, b, c), Singh et al. (2009) and Singh and Kumar (2011) suggested estimators using known population parameters of an auxiliary variable. But sometimes it is better to use information on two auxiliary variables rather than one auxiliary variable for the estimation of finite population mean. Sharma and Singh (2014) and Lu and Yan (2014) studied the properties of some estimators using two auxiliary variables, Singh and kumar (2012) suggested improved estimators of population mean using two auxiliary variables in stratified random sampling. This paper discusses the problem of estimation of finite population mean using information on two auxiliary variates.
Consider a finite population of size N and is divided into L strata with N_h elements in the h^{th} stratum such that $N=\sum_{h=1}^{L} N_h$. Using simple random sampling without replacement we select a sample of size n_h from each stratum

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such that $n = \sum_{h=1}^{L} n_h$, where n_h is the stratum sample size. Let y_{hi} , x_{hi} and z_{hi} be the values of the study variable Y and auxiliary variables X, Z respectively in the h^{th} stratum.

Let $W_h = N_h / N$ be the stratum weight, $f_h = n_h / N_h$ sampling fraction, $\gamma_h = (1-f_h)/n_h$, $\bar{y}_h = \frac{1}{n_h}\sum_{i=1}^{n_h} y_{hi}$, $\bar{x}_h = \frac{1}{n_h}\sum_{i=1}^{n_h} x_{hi}$, $\bar{z}_h = \frac{1}{n_h}\sum_{i=1}^{n_h} z_{hi}$, the y, x and z sample means, $\bar{Y}_h = \frac{1}{N_h}\sum_{i=1}^{N_h} y_{hi}$, $\bar{X}_h = \frac{1}{N_h}\sum_{i=1}^{N_h} x_{hi}$, $\bar{Z}_h = \frac{1}{N_h}\sum_{i=1}^{N_h} z_{hi}$, the Y, X and Z population means respectively. Assume that $\bar{X} = \sum_{h=1}^{L} W_h \bar{X}_h$ and $\bar{Z} = \sum_{h=1}^{L} W_h \bar{Z}_h$ are known and we want to estimate $\bar{Y}_h = \sum_{h=1}^{L} W_h \bar{Y}_h$ by using information on auxiliary variables X and Z.

When we have information on two auxiliary variables then usual regression estimator is defined as

$$\bar{y}_{lr} = \bar{y}_{st} + b_1\left(\bar{X} - \bar{x}_{st}\right) + b_2\left(\bar{Z} - \bar{z}_{st}\right) \tag{1.1}$$

Where $b_1 = \frac{s_{yx}}{s_x^2}$ and $b_2 = \frac{s_{yz}}{s_z^2}$. s_x^2 and s_x^2 are the sample variances of x and z respectively, s_{yz} and s_{yz} are the sample covariance's between y, x and between y, z respectively.

The MSE expression of estimator (1.1) is given as

$$MSE\left(\bar{y}_{lr}\right) = \sum W_h^2 \gamma_h S_{yh}^2 \left(1 - \rho_{yxh}^2 - \rho_{yzh}^2 + 2\rho_{yxh}\rho_{yzh}\rho_{xzh}\right) \tag{1.2}$$

where , $S_{yh}^2 = \frac{\sum_{i=1}^{N_h}\left(y_{hi} - \bar{Y}_h\right)}{N_h - 1}$, $S_{xh}^2 = \frac{\sum_{i=1}^{N_h}\left(x_{hi} - \bar{X}_h\right)}{N_h - 1}$, $S_{zh}^2 = \frac{\sum_{i=1}^{N_h}\left(z_{hi} - \bar{Z}_h\right)}{N_h - 1}$, $S_{xyh} = \frac{\sum_{i=1}^{N_h}\left(x_{hi} - \bar{X}_h\right)\left(y_{hi} - \bar{Y}_h\right)}{N_h - 1}$,

$S_{zyh} = \frac{\sum_{i=1}^{N_h}\left(z_{hi} - \bar{Z}_h\right)\left(y_{hi} - \bar{Y}_h\right)}{N_h - 1}$, $\rho_{yxh} = \frac{S_{yxh}}{S_{yh}S_{xh}}$, $\rho_{yzh} = \frac{S_{yzh}}{S_{yh}S_{zh}}$, $\rho_{xzh} = \frac{S_{xzh}}{S_{xh}S_{zh}}$

2. PROPOSED CLASSES OF ESTIMATOR

Motivated by Swain (2013), we propose a class of generalized modified type ratio estimator using two auxiliary variables in stratified sampling as

$$t_g = \bar{y}_{st}\left[\alpha_1\left(\frac{\bar{X}}{\bar{x}_{st}}\right)^{g_1} + (1-\alpha_1)\left(\frac{\bar{x}_{st}}{\bar{X}}\right)^{h_1}\right]^{\delta_1}\left[\alpha_2\left(\frac{\bar{Z}}{\bar{z}_{st}}\right)^{g_2} + (1-\alpha_2)\left(\frac{\bar{z}_{st}}{\bar{Z}}\right)^{h_2}\right]^{\delta_2} \tag{2.1}$$

Where α_1, α_2, g_1, g_2, h_1, h_2, δ_1 and δ_2 are real constants to be determined suitably.

Following Swain (2013), we propose another generalized class of difference-cum-ratio estimator, defined as

$$t_s = \left[\bar{y}_{st} + \lambda\left(\bar{X} - \bar{x}_{st}\right)\right]\left[\alpha\left(\frac{\bar{Z}}{\bar{z}_{st}}\right)^{g} + (1-\alpha)\left(\frac{\bar{z}_{st}}{\bar{Z}}\right)^{h}\right]^{\delta} \tag{2.2}$$

Where λ, α, g, h and δ are real constants to be determined suitably and $0 < \alpha < 1$.

We propose another estimator t_N as

$$t_N = \bar{y}_{st}\left[w_1\left(\frac{\bar{x}_{st}}{\bar{X}}\right)^{\alpha} \exp\left\{\frac{\eta_1\left(\bar{X}-\bar{x}_{st}\right)}{\eta_1\left(\bar{X}+\bar{x}_{st}\right)+2\lambda_1}\right\} + w_2\left(\frac{\bar{z}_{st}}{\bar{Z}}\right)^{\beta} \exp\left\{\frac{\eta_2\left(\bar{Z}-\bar{z}_{st}\right)}{\eta_2\left(\bar{Z}+\bar{z}_{st}\right)+2\lambda_2}\right\}\right] \tag{2.3}$$

Where w_1, w_2, α, β, η_1, η_2, λ_1 and λ_2 are real constants to be determined suitably and $w_1 + w_2 \neq 1$.

To obtain the bias and MSE of the estimators t_g, t_s and t_N to the first degree of approximation, we write

$$e_0 = \frac{\bar{y}_{st} - \bar{Y}}{\bar{Y}},\quad e_1 = \frac{\bar{x}_{st} - \bar{X}}{\bar{X}},\quad e_2 = \frac{\bar{z}_{st} - \bar{Z}}{\bar{Z}}$$

Such that, $E(e_i) = 0$; $i = 0, 1, 2.$

Also,

$$E\left(e_0^2\right) = \frac{\sum_{h=1}^{L} W_h^2 \gamma_h S_{yh}^2}{\bar{Y}^2} = V_0,\quad E\left(e_1^2\right) = \frac{\sum_{h=1}^{L} W_h^2 \gamma_h S_{xh}^2}{\bar{X}^2} = V_1,\quad E\left(e_2^2\right) = \frac{\sum_{h=1}^{L} W_h^2 \gamma_h S_{zh}^2}{\bar{Z}^2} = V_2,$$

$$E\left(e_0 e_1\right) = \frac{\sum_{h=1}^{L} W_h^2 \gamma_h S_{yxh}}{\bar{Y}\bar{X}} = V_3,\ E\left(e_0 e_2\right) = \frac{\sum_{h=1}^{L} W_h^2 \gamma_h S_{yzh}}{\bar{Y}\bar{Z}} = V_4,\quad E\left(e_1 e_2\right) = \frac{\sum_{h=1}^{L} W_h^2 \gamma_h S_{xzh}}{\bar{X}\bar{Z}} = V_5$$

Expressing (2.1) in terms of e's we have

$$t_g = \bar{Y}\left(1+e_0\right)\left[1+\left(A_g e_1 + B_g \frac{e_1^2}{2}\right)\right]^{\delta_1}\left[1+\left(C_g e_2 + D_g \frac{e_2^2}{2}\right)\right]^{\delta_2} \tag{2.4}$$

We assume that $\left|\left(A_g e_1 + B_g \frac{e_1^2}{2}\right)\right| < 1$ and $\left|\left(C_g e_2 + D_g \frac{e_2^2}{2}\right)\right| < 1$ so that the term $\left[1+\left(A_g e_1 + B_g \frac{e_1^2}{2}\right)\right]^{\delta_1}$ and $\left[1+\left(C_g e_2 + D_g \frac{e_2^2}{2}\right)\right]^{\delta_2}$ can be expanded.

Where

$$A_g = h_1 - \alpha_1\left(g_1 + h_1\right)$$
$$B_g = \alpha_1 g_1\left(g_1 + 1\right) + \left(1-\alpha_1\right)h_1\left(h_1 - 1\right)$$
$$C_g = h_2 - \alpha_2\left(g_2 + h_2\right)$$
$$D_g = \alpha_2 g_2\left(g_2 + 1\right) + \left(1-\alpha_2\right)h_2\left(h_2 - 1\right)$$

Expanding the right hand side of (2.4) up to the first order of approximation, we have

$$t_g - \bar{Y} = \bar{Y}\left[e_0 + A_g\delta_1 e_1 + C_g\delta_2 e_2 + \left(B_g\delta_1 + \delta_1\left(\delta_1 - 1\right)A_g^2\right)\frac{e_1^2}{2} + \left(D_g\delta_2 + \delta_2\left(\delta_2 - 1\right)C_g^2\right)\frac{e_2^2}{2} + A_g\delta_1 e_0 e_1 + C_g\delta_2 e_0 e_2 + A_g C_g \delta_1\delta_2 e_1 e_2\right] \tag{2.5}$$

Taking expectation of both sides of (2.5), we get the Bias expression of t_g as

$$\mathrm{Bias}(t_g) = \bar{Y}\left[\frac{1}{2}\left(B_g + \left(\delta_1 - 1\right)A_g^2\right)\delta_1 V_1 + \frac{1}{2}\left(D_g + \left(\delta_2 - 1\right)C_g^2\right)\delta_2 V_2 + A_g\delta_1 V_3 + C_g\delta_2 V_4 + A_g C_g \delta_1\delta_2 V_5\right] \tag{2.6}$$

Squaring both sides of (2.5) and neglecting the terms e's having power greater than two, we have

$$(t_g - \bar{Y})^2 = \bar{Y}^2\left[e_0^2 + A_g^2\delta_1^2 e_1^2 + C_g^2\delta_2^2 e_2^2 + 2A_g\delta_1 e_0 e_1 + 2C_g\delta_2 e_0 e_2 + 2A_g C_g \delta_1\delta_2 e_1 e_2\right] \qquad (2.7)$$

Taking expectations of both sides of (2.7), we get the MSE of t_g given by

$$\begin{aligned}MSE(t_g) = \bar{Y}^2 \big[&V_0 + \alpha_1^2 A_1 + \alpha_2^2 A_4 - 2\alpha_1 A_3 - 2\alpha_2 A_5 + 2\alpha_1\alpha_2 A_2 + h_1^2\delta_1^2 V_1 + h_2^2\delta_2^2 V_2 + 2h_1\delta_1 V_3 \\ &+ 2h_2\delta_2 V_4 + 2h_1 h_2 \delta_1\delta_2 V_5\big]\end{aligned} \qquad (2.8)$$

Where

$$A_1 = \delta_1^2(g_1 + h_1)^2 V_1$$
$$A_2 = \delta_1\delta_2(g_1 + h_1)(g_2 + h_2)V_5$$
$$A_3 = \delta_1(g_1 + h_1)(\delta_1 h_1 V_1 + V_3 + \delta_2 h_2 V_5)$$
$$A_4 = \delta_2^2(g_2 + h_2)^2 V_2$$
$$A_5 = \delta_2(g_2 + h_2)(\delta_1 h_1 V_5 + V_4 + \delta_2 h_2 V_2)$$

Differentiating with respect to α_1 and α_2 and equating to zero, we get the optimum value of α_1 amd α_2 as

$$\alpha_1(opt) = \frac{A_3 A_4 - A_2 A_5}{A_1 A_4 - A_2^2}$$

$$\alpha_2(opt) = \frac{A_1 A_5 - A_2 A_3}{A_1 A_4 - A_2^2}$$

Similarly following the above procedures we get the Bias and MSE expressions of estimator t_s and t_N as

$$Bias(t_s) = \bar{Y}\left[\left(B_s + \frac{(\delta - 1)}{2}A_s^2\right)\delta V_2 + \delta A_s V_4\right] - \delta\lambda A_s \bar{X} V_5 \qquad (2.9)$$

Where

$$A_s = \alpha g - (1 - \alpha)h$$
$$B_s = \frac{1}{2}\{\alpha g(g + 1) - (1 - \alpha)h(h - 1)\}$$

$$Bias(t_N) = \bar{Y}[w_1\{1 - A_N V_3 + B_N V_1\} + w_2\{1 - C_N V_4 + D_N V_2\} - 1] \qquad (2.10)$$

Where

$$A_N = \alpha - \gamma_1$$
$$B_N = \frac{3}{2}\gamma_1^2 - \alpha\gamma_1 + \frac{\alpha(\alpha - 1)}{2}$$
$$C_N = \beta - \gamma_2$$
$$D_N = \frac{3}{2}\gamma_2^2 - \beta\gamma_2 + \frac{\beta(\beta - 1)}{2}$$

and MSE of the estimator t_s

$$\begin{aligned}MSE(t_s) = \bar{Y}^2\big[&V_0 + \delta^2\{\alpha^2 g^2 + (1 - \alpha)^2 h^2 - 2\alpha(1 - \alpha)gh\}V_2 + 2\delta\{\alpha g - (1 - \alpha)h\}V_4\big] \\ &+ \lambda^2\bar{X}^2 V_1 - 2\lambda\bar{X}\bar{Y}\{V_3 + \delta(\alpha g - (1 - \alpha)h)V_5\}\end{aligned} \qquad (2.11)$$

and the optimum value of α and λ are given as

$$\alpha(opt) = \frac{V_1(V_4 - \delta h V_2) - V_5(V_3 - \delta h V_5)}{\delta(g + h)(V_5^2 - V_1 V_2)}$$

$$\lambda(\text{opt}) = \frac{\bar{Y}\{V_5(V_4 - \delta h V_2) - V_2(V_3 - \delta h V_5)\}}{\bar{X}(V_5^2 - V_1 V_2)}$$

$$MSE(t_N) = \bar{Y}^2\left[1 + w_1^2 B_1 + w_2^2 B_2 - 2w_1 B_3 - 2w_2 B_4 + 2w_1 w_2 B_5\right] \tag{2.12}$$

where

$$B_1 = 1 + V_0 + V_1(A_N^2 + 2B_N) - 4A_N V_3$$
$$B_2 = 1 + V_0 + V_2(C_N^2 + 2D_N) - 4C_N V_4$$
$$B_3 = 1 - A_N V_3 + B_N V_1$$
$$B_4 = 1 - C_N V_4 + D_N V_2$$
$$B_5 = 1 + V_0 + B_N V_1 + D_N V_2 - 2A_N V_3 - 2C_N V_4 + AC_N V_5$$

and the optimum value of w_1 and w_2 are given as

$$w_1(\text{opt}) = \frac{B_2 B_3 - B_4 B_5}{B_1 B_2 - B_5^2}$$
$$w_2(\text{opt}) = \frac{B_1 B_4 - B_3 B_5}{B_1 B_2 - B_5^2}$$

Table 2.1: Set of estimators generated from t_g

g_1	h_1	g_2	h_2	δ_1	δ_2	Estimator
1	1	1	1	1	1	$t_{g1} = \bar{y}_{st}\left[\alpha_1\left(\frac{\bar{X}}{\bar{x}_{st}}\right) + (1-\alpha_1)\left(\frac{\bar{x}_{st}}{\bar{X}}\right)\right]\left[\alpha_2\left(\frac{\bar{Z}}{\bar{z}_{st}}\right) + (1-\alpha_2)\left(\frac{\bar{z}_{st}}{\bar{Z}}\right)\right]$
1	0	1	0	1	1	$t_{g2} = \bar{y}_{st}\left[\alpha_1\left(\frac{\bar{X}}{\bar{x}_{st}}\right) + (1-\alpha_1)\right]\left[\alpha_2\left(\frac{\bar{Z}}{\bar{z}_{st}}\right) + (1-\alpha_2)\right]$
0	1	0	1	1	1	$t_{g3} = \bar{y}_{st}\left[\alpha_1 + (1-\alpha_1)\left(\frac{\bar{x}_{st}}{\bar{X}}\right)\right]\left[\alpha_2 + (1-\alpha_2)\left(\frac{\bar{z}_{st}}{\bar{Z}}\right)\right]$
1	1	1	1	-1	-1	$t_{g4} = \frac{\bar{y}_{st}}{\left[\alpha_1\left(\frac{\bar{X}}{\bar{x}_{st}}\right) + (1-\alpha_1)\left(\frac{\bar{x}_{st}}{\bar{X}}\right)\right]\left[\alpha_2\left(\frac{\bar{Z}}{\bar{z}_{st}}\right) + (1-\alpha_2)\left(\frac{\bar{z}_{st}}{\bar{Z}}\right)\right]}$
1	0	1	0	-1	-1	$t_{g5} = \frac{\bar{y}_{st}}{\left[\alpha_1\left(\frac{\bar{X}}{\bar{x}_{st}}\right) + (1-\alpha_1)\right]\left[\alpha_2\left(\frac{\bar{Z}}{\bar{z}_{st}}\right) + (1-\alpha_2)\right]}$
0	1	0	1	-1	-1	$t_{g6} = \frac{\bar{y}_{st}}{\left[\alpha_1 + (1-\alpha_1)\left(\frac{\bar{x}_{st}}{\bar{X}}\right)\right]\left[\alpha_2 + (1-\alpha_2)\left(\frac{\bar{z}_{st}}{\bar{Z}}\right)\right]}$

1	1	1	1	1	-1	$t_{g7} = \bar{y}_{st} \dfrac{\left[\alpha_1\left(\dfrac{\bar{X}}{\bar{x}_{st}}\right)+(1-\alpha_1)\left(\dfrac{\bar{x}_{st}}{\bar{X}}\right)\right]}{\left[\alpha_2\left(\dfrac{\bar{Z}}{\bar{z}_{st}}\right)+(1-\alpha_2)\left(\dfrac{\bar{z}_{st}}{\bar{Z}}\right)\right]}$
1	0	1	0	1	-1	$t_{g8} = \bar{y}_{st} \dfrac{\left[\alpha_1\left(\dfrac{\bar{X}}{\bar{x}_{st}}\right)+(1-\alpha_1)\right]}{\left[\alpha_2\left(\dfrac{\bar{Z}}{\bar{z}_{st}}\right)+(1-\alpha_2)\right]}$
0	1	0	1	1	-1	$t_{g9} = \bar{y}_{st} \dfrac{\left[\alpha_1+(1-\alpha_1)\left(\dfrac{\bar{x}_{st}}{\bar{X}}\right)\right]}{\left[\alpha_2+(1-\alpha_2)\left(\dfrac{\bar{z}_{st}}{\bar{Z}}\right)\right]}$

Table 2.2: Set of estimators generated from t_s

g	h	δ	Estimator
1	1	1	$t_{s1} = \left[\bar{y}_{st} + \lambda(\bar{X}-\bar{x}_{st})\right]\left[\alpha\left(\dfrac{\bar{Z}}{\bar{z}_{st}}\right)+(1-\alpha)\left(\dfrac{\bar{z}_{st}}{\bar{Z}}\right)\right]$
1	0	1	$t_{s2} = \left[\bar{y}_{st} + \lambda(\bar{X}-\bar{x}_{st})\right]\left[\alpha\left(\dfrac{\bar{Z}}{\bar{z}_{st}}\right)+(1-\alpha)\right]$
0	1	1	$t_{s3} = \left[\bar{y}_{st} + \lambda(\bar{X}-\bar{x}_{st})\right]\left[\alpha+(1-\alpha)\left(\dfrac{\bar{z}_{st}}{\bar{Z}}\right)\right]$
1	1	-1	$t_{s4} = \dfrac{\bar{y}_{st} + \lambda(\bar{X}-\bar{x}_{st})}{\alpha\left(\dfrac{\bar{Z}}{\bar{z}_{st}}\right)+(1-\alpha)\left(\dfrac{\bar{z}_{st}}{\bar{Z}}\right)}$
1	0	-1	$t_{s5} = \dfrac{\bar{y}_{st} + \lambda(\bar{X}-\bar{x}_{st})}{\alpha\left(\dfrac{\bar{Z}}{\bar{z}_{st}}\right)+(1-\alpha)}$
0	1	-1	$t_{s6} = \dfrac{\bar{y}_{st} + \lambda(\bar{X}-\bar{x}_{st})}{\alpha+(1-\alpha)\left(\dfrac{\bar{z}_{st}}{\bar{Z}}\right)}$

Table 2.3: Set of estimators generated from t_N

η_1	η_2	λ_1	λ_2	α	β	Estimator
1	1	1	1	1	1	$t_{N1}=\bar{y}_{st}\left[w_1\left(\frac{\bar{x}_{st}}{\bar{X}}\right)\exp\left\{\frac{(\bar{X}-\bar{x}_{st})}{(\bar{X}+\bar{x}_{st})+2}\right\}+w_2\left(\frac{\bar{z}_{st}}{\bar{Z}}\right)\exp\left\{\frac{(\bar{Z}-\bar{z}_{st})}{(\bar{Z}+\bar{z}_{st})+2}\right\}\right]$
1	1	1	1	-1	1	$t_{N2}=\bar{y}_{st}\left[w_1\left(\frac{\bar{X}}{\bar{x}_{st}}\right)\exp\left\{\frac{(\bar{X}-\bar{x}_{st})}{(\bar{X}+\bar{x}_{st})+2}\right\}+w_2\left(\frac{\bar{z}_{st}}{\bar{Z}}\right)\exp\left\{\frac{(\bar{Z}-\bar{z}_{st})}{(\bar{Z}+\bar{z}_{st})+2}\right\}\right]$
1	1	1	1	1	-1	$t_{N3}=\bar{y}_{st}\left[w_1\left(\frac{\bar{x}_{st}}{\bar{X}}\right)\exp\left\{\frac{(\bar{X}-\bar{x}_{st})}{(\bar{X}+\bar{x}_{st})+2}\right\}+w_2\left(\frac{\bar{Z}}{\bar{z}_{st}}\right)\exp\left\{\frac{(\bar{Z}-\bar{z}_{st})}{(\bar{Z}+\bar{z}_{st})+2}\right\}\right]$
1	1	1	1	1	0	$t_{N4}=\bar{y}_{st}\left[w_1\left(\frac{\bar{x}_{st}}{\bar{X}}\right)\exp\left\{\frac{(\bar{X}-\bar{x}_{st})}{(\bar{X}+\bar{x}_{st})+2}\right\}+w_2\exp\left\{\frac{(\bar{Z}-\bar{z}_{st})}{(\bar{Z}+\bar{z}_{st})+2}\right\}\right]$
1	1	1	1	0	1	$t_{N5}=\bar{y}_{st}\left[w_1\exp\left\{\frac{(\bar{X}-\bar{x}_{st})}{(\bar{X}+\bar{x}_{st})+2}\right\}+w_2\left(\frac{\bar{z}_{st}}{\bar{Z}}\right)\exp\left\{\frac{(\bar{Z}-\bar{z}_{st})}{(\bar{Z}+\bar{z}_{st})+2}\right\}\right]$
$\beta_2(x)$	$\beta_2(z)$	1	1	-1	1	$t_{N6}=\bar{y}_{st}\left[w_1\left(\frac{\bar{X}}{\bar{x}_{st}}\right)\exp\left\{\frac{\beta_2(x)(\bar{X}-\bar{x}_{st})}{\beta_2(x)(\bar{X}+\bar{x}_{st})+2}\right\}+w_2\left(\frac{\bar{z}_{st}}{\bar{Z}}\right)\exp\left\{\frac{\beta_2(z)(\bar{Z}-\bar{z}_{st})}{\beta_2(z)(\bar{Z}+\bar{z}_{st})+2}\right\}\right]$
$\beta_2(x)$	$\beta_2(z)$	1	1	1	-1	$t_{N7}=\bar{y}_{st}\left[w_1\left(\frac{\bar{x}_{st}}{\bar{X}}\right)\exp\left\{\frac{\beta_2(x)(\bar{X}-\bar{x}_{st})}{\beta_2(x)(\bar{X}+\bar{x}_{st})+2}\right\}+w_2\left(\frac{\bar{Z}}{\bar{z}_{st}}\right)\exp\left\{\frac{\beta_2(z)(\bar{Z}-\bar{z}_{st})}{\beta_2(z)(\bar{Z}+\bar{z}_{st})+2}\right\}\right]$
$\beta_2(x)$	$\beta_2(z)$	$\beta_2(x)$	$\beta_2(z)$	-1	1	$t_{N8}=\bar{y}_{st}\left[w_1\left(\frac{\bar{X}}{\bar{x}_{st}}\right)\exp\left\{\frac{\beta_2(x)(\bar{X}-\bar{x}_{st})}{\beta_2(x)(\bar{X}+\bar{x}_{st})+2\beta_2(x)}\right\}+w_2\left(\frac{\bar{z}_{st}}{\bar{Z}}\right)\exp\left\{\frac{\beta_2(z)(\bar{Z}-\bar{z}_{st})}{\beta_2(z)(\bar{Z}+\bar{z}_{st})+2\beta_2(z)}\right\}\right]$
$\beta_2(x)$	$\beta_2(z)$	$\beta_2(x)$	$\beta_2(z)$	1	-1	$t_{N9}=\bar{y}_{st}\left[w_1\left(\frac{\bar{x}_{st}}{\bar{X}}\right)\exp\left\{\frac{\beta_2(x)(\bar{X}-\bar{x}_{st})}{\beta_2(x)(\bar{X}+\bar{x}_{st})+2\beta_2(x)}\right\}+w_2\left(\frac{\bar{Z}}{\bar{z}_{st}}\right)\exp\left\{\frac{\beta_2(z)(\bar{Z}-\bar{z}_{st})}{\beta_2(z)(\bar{Z}+\bar{z}_{st})+2\beta_2(z)}\right\}\right]$

1	1	$\beta_2(x)$	$\beta_2(z)$	-1	1	$t_{N10} = \bar{y}_{st}\left[w_1 \left(\frac{\bar{X}}{\bar{x}_{st}} \right) \exp\left\{ \frac{(\bar{X} - \bar{x}_{st})}{(\bar{X} + \bar{x}_{st}) + 2\beta_2(x)} \right\} + w_2 \left(\frac{\bar{z}_{st}}{\bar{Z}} \right) \exp\left\{ \frac{(\bar{Z} - \bar{z}_{st})}{(\bar{Z} + \bar{z}_{st}) + 2\beta_2(z)} \right\} \right]$
$\beta_2(x)$	$\beta_2(z)$	0	0	-1	1	$t_{N11} = \bar{y}_{st}\left[w_1 \left(\frac{\bar{X}}{\bar{x}_{st}} \right) \exp\left\{ \frac{\beta_2(x)(\bar{X} - \bar{x}_{st})}{\beta_2(x)(\bar{X} + \bar{x}_{st})} \right\} + w_2 \left(\frac{\bar{z}_{st}}{\bar{Z}} \right) \exp\left\{ \frac{\beta_2(z)(\bar{Z} - \bar{z}_{st})}{\beta_2(z)(\bar{Z} + \bar{z}_{st})} \right\} \right]$

Similarly we can construct many more estimators but here we show only few good estimators.

Where $\beta_2(x) = \sum_{h=1}^{L} W_h \beta_2(x_h)$ and $\beta_2(z) = \sum_{h=1}^{L} W_h \beta_2(z_h)$.

3. EMPIRICAL STUDY

To check the efficiency of the proposed estimators empirically in comparison to other estimators here we consider a natural population data set and description of the populations are given below:
Population [Source: Koyuncu and Kadilar (2009)]
Y: Number of teachers,
X: Number of students,
Z: Number of classes in both primary and secondary school.

Table 3.1: Variance/MSEs and PRE of estimators

N_1 =127	N_2 =117	N_3 =103
N_4 =170	N_5 =205	N_6 =201
n_1 =31	n_2 =21	n_3 =29
n_4 =38	n_5 =22	n_6 =39
$\bar{Y}_1$ =703.74	$\bar{Y}_2$ =413	$\bar{Y}_3$ =573.17
$\bar{Y}_4$ =424.66	$\bar{Y}_5$ =267.03	$\bar{Y}_6$ =393.84
$\bar{X}_1$ =20804.59	$\bar{X}_2$ =9211.79	$\bar{X}_3$ =14309.30
$\bar{X}_4$ =9478.85	$\bar{X}_5$ =5569.95	$\bar{X}_6$ =12997.59
$\bar{Z}_1$ =498.28	$\bar{Z}_2$ =318.33	$\bar{Z}_3$ =431.36

$\bar{Z}_4$ =311.32 $\bar{Z}_5$ =227.20 $\bar{Z}_6$ =313.71

S_{y1} =883.835 S_{y2} =644 S_{y3} =1033.467

S_{y4} =810.585 S_{y5} =403.654 S_{y6} =711.723

S_{x1} =30486.751 S_{x2} =15180.760 S_{x3} =27549.697

S_{x4} =18218.931 S_{x5} =8997.776 S_{x6} =23094.141

S_{z1} =555.5816 S_{z2} =365.4576 S_{z3} =612.9509

S_{z4} =458.0282 S_{z5} =260.8511 S_{z6} =397.0481

S_{yx1} =25237153.52 S_{yx2} =9747942.85 S_{yx3} =28294397.04

S_{yx4} =1452885.53 S_{yx5} =3393591.75 S_{yx6} =15864573.97

S_{yz1} =480688.2 S_{yz2} =230092.8 S_{yz3} =623019.3

S_{yz4} =36493.4 S_{yz5} =101539 S_{yz6} =277696.1

S_{xz1} =15914648 S_{xz2} =5379190 S_{xz3} =164900674.56

S_{xz4} =8041254 S_{xz5} =214457 S_{xz6} =8857729

$\beta_2(x_1)$=4.593 $\beta_2(x_2)$=18.543 $\beta_2(x_3)$=15.446

$\beta_2(x_4)$=10.162 $\beta_2(x_5)$=21.947 $\beta_2(x_6)$=23.114

$\beta_2(y_1)$=2.158 $\beta_2(y_2)$=16.392 $\beta_2(y_3)$=14.979

$\beta_2(y_4)$=12.167 $\beta_2(y_5)$=21.008 $\beta_2(y_6)$=20.254

$\beta_2(z_1)$=2.314 $\beta_2(z_2)$=11.190 $\beta_2(z_3)$=10.786

$\beta_2(z_4)$=8.624 $\beta_2(z_5)$=9.720 $\beta_2(z_6)$=14.406

Table 3.1 shows the Variance/MSEs of different estimators along with percentage relative efficiency (PRE) of different estimators with respect to $\bar{y}_{st}$.

where, $PRE(.) = \dfrac{V(\bar{y}_{st})}{MSE(.)} \times 100$

Estimators	Variance/MSEs	PRE with respect to $\bar{y}_{st}$
$\bar{y}_{st}$	2229.2662	100
t_{reg}	928.8951	239.9912
t_{gi} (i=1…9)	928.8951	239.9912

t_{si} (i=1…6)	928.8951	239.9912
t_{N1}	1018.8320	218.8061
t_{N2}	807.2172	276.1668
t_{N3}	598.2903	372.6061
t_{N4}	**554.1423**	**402.2913**
t_{N5}	736.2893	302.7704
t_{N6}	808.1719	275.8406
t_{N7}	598.3615	372.5617
t_{N8}	807.2172	276.1668
t_{N9}	598.2903	372.6061
t_{N10}	805.0066	276.9252
t_{N11}	808.8981	275.5929

Remark: The values of α, β, λ_1, λ_2, η_1 and η_2 are shown in the table 2.3 for the MSE correspond to the estimators t_{N1},……., t_{N11}.

4. CONCLUSION

In this paper, we suggests some generalized classes of modified estimators whose members are given in table 2.1, 2.2 and 2.3 respectively for finite population mean of the study variable utilizing the information on two auxiliary variables in stratified random sampling. From the above table we conclude that the efficiency of estimators t_g and t_s are same and equal to usual regression estimator using two auxiliary variables. The estimator t_{N4} of the proposed class of estimators t_N has minimum MSE among all estimators generated from t_N as well as other existing and proposed classes of estimators. Thus the estimator t_{N4} is most efficient than other estimators consider here under given conditions.

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